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COMPARATIVE PROFIT ANALYSIS OF TWO RELIABILITY MODELS VARYING MAINTENANCE POLICIES FOR STANDBY UNIT

Anju Dhall*

ABSTRACT

This paper analysis a stochastic system with two different types of maintenance policies for the standby unit before getting it into operation. In Model1, the standby unit undergoes for maintenance with some probability when it is not operable at the failure of operative unit. While in Model2, server first inspects the standby unit to see the feasibility of its operation at the failure of operating unit. And, if standby unit is not found fit for operation, then the server starts its maintenance immediately. A single server is available instantly to provide the services like repairs of hardware, software components and preventive maintenance. The system works as new after repair. All the key reliability characteristics and other parameters of the system like mean time to system failure(MTSF), availability of the system, busy period of the server, the number of visits of the server and cost-benefit of the system are derived by RPGT.

Keywords: *Base –State, Capacity Factor, Fuzziness Measure, RPGT.*

Introduction

A lot of research work has been carried out on reliability modeling of stochastic systems in view of their more natural existence in real life and may be more appropriate in practicein the past few decades. The researchers and scientists got success in some extent in improving the performance of the individual components that constitutes the system. And, the system with standby(s) has been observed as the effective one for providing better services for a long time. Therefore, several research papers on reliability measures of redundant systems have been written by the scholars including Malik and Kumar (2012), Bansalritu and Goel (2016) and Pradeepchaudhary et al. (2022).But in most of these studies it is assumed that unit in standby mode cannot deteriorate. Practically, this assumption seems to be unrealistic when a system has to work in varying environmental conditions. The standby unit may deteriorate due to many reasons such as non-functionality of the unit for a long time and by erosion. On the other hand, readiness of spares is of vital significance as these can be brought into operation at the failure of operating unit. Thus, in such a situation, maintenance of the standby unit becomes necessary before getting it into operation and this fact can be revealed by inspection. Recently, Malik and dhall(2014) evaluated performance measures of cold standby systems with maintenance and repair.

In view of such practical situations in mind, the aim of the present study is to analyze reliability models of a stochastic system of two identical units - one unit is initially operative and the other is kept as spare in cold standby. The unit has two modes – operative and failure. A single server is available immediately for conducting maintenance and other repair activities. The unit in standby mode may

* Department of Mathematics, G.G.D.S.D. College, Palwal, Haryana, India.

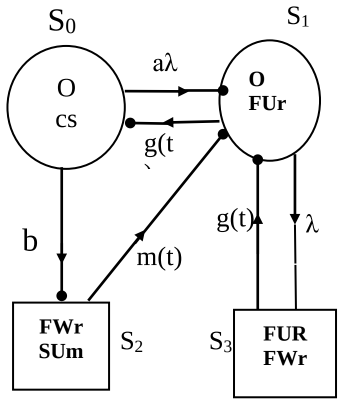
deteriorate. Two different maintenance policies for the standby unit have been adopted before getting it into operation. In first policy, a model is developed in which standby unit undergoes for maintenance with some probability when it is not operable at the failure of operative unit. According to second policy, server first inspects the standby unit to see the feasibility of its operation at the failure of operating unit. And, if standby unit is not found fit for operation, then the server starts its maintenance immediately. However, only repair of the operative unit is done by the server. The unit works as new after maintenance and repair. The failure time of the unit follows negative exponential distribution where as the distributions of maintenance, inspection and repair times are taken as arbitrary with different probability density functions. All random variables are statistically independent. Switch devices are perfect. The expressions for several reliability measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to inspection and maintenance, busy period of the server due to repair, expected number of visits of the server and profit function have been derived in steady state using semi-Markov process and regenerative point technique. The behavior of some important performance measures have been examined graphically giving particular values to various parameters and costs. Profit comparison of the system models has been made to see the utility of the maintenance policies.

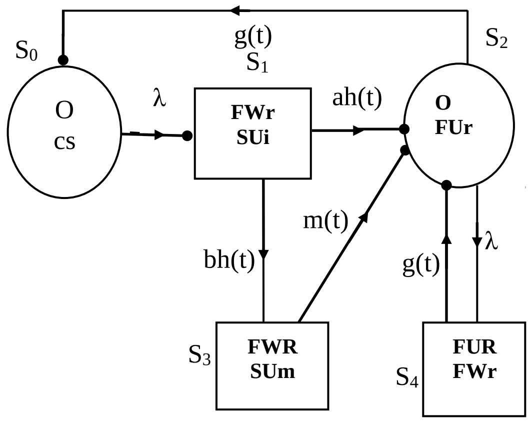
Notations

$\mathrm{O/Cs}$	The unit is in operative/ cold standby mode
λ	Constant failure rate of the unit.
$\mathrm{a/b}$	The probability that standby unit is found fit for operation/non-operable.
SU_i/SU_m	The rate Standby unit under inspection/maintenance.
$\mathrm{g(t)/G(t)}$	pdf/cdf of repair time .
$\mathrm{h(t)/H(t))}$	pdf/cdf of inspection time of the unit.
$\mathrm{m(t)/M(t)}$	pdf/cdf of preventive maintenance time.
FUr/FWr	The unit is failed and under/waiting for repair continuously from previous state.
FUi/FUI	The unit is failed and under/waiting for inspection continuously from previous state.
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of direct transition time from a regenerative state regenerative state(0,t].
$q_{ij.k}(t)/Q_{ij.k}(t)$	pdf/cdf of first passage time for a regenerative state S_i to regenerative state S_j or to failed state S_j visiting state S_k once in (0,t]
$W_i(t)$	Probability that the server is busy in state S_i up to time without making transition to any other regenerative state or returning to the same via one or more regenerative states.
$M_i(t)$	Probability that the system is up initially in state $S_i \in E$ is up at the time "t" without visiting to any other regenerative state.
μ_i	The mean sojourn time spent in state $S_i \in E$ before transition to any other state.
μ_i'	The total unconditional time spent in state before transition to any other regenerative state given that the system entered regenerative state i at time t=0
f_i	Fuzziness measure of the i-state
n_i	Expected time spend while doing a job, given that the system entered regenerative state i at time t=0
$V_{kk}/\overline{V_{kk}}$	Transition probability factor of reachable state k of the $k\ cycle/\overline{k\ cycle}$
$i \xrightarrow{r} j$	R^{th} directed simple path from i state to j state,r take +ve integral values from i state to j state
$\xi \xrightarrow{sff} i$	A directed simple failure free path from ξ state to i state
m_{ij}	Contribution to mean sojourn time in state S_i when system transits directly to state S_j(S_i,$S_j \in$ E) $\mu_i = \sum m_{ij}$ so that $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$
Ⓢ/©	Laplace Stieljes convolution/ Laplace convolution
$\sim/*$	Symbol for Laplace Stieljes transform (LST)/ Laplace transform (LT
$'$	Symbol for derivative of the function
cycles	$(i,j,k) = (i,j)(j,k)$

The possible transition states of the system models are shown respectively in figures 1 and 2.

State Transition Diagrams





Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$$ as

For Model 1

$p_{01} = a, p_{02} = b, p_{10} = g^*(\lambda), p_{13} = 1 - g^*(\lambda),, p_{21} = m^*(0), p_{31} = g^*(0) \cdots (1)$

It can be verified that

$p_{01} + p_{02} = 1; p_{10} + p_{13} = 1; p_{10} + p_{11.3} = 1; p_{21} = 1; p_{31} = 1 \qquad \cdots (2)$

For Model 2

$p_{01} = 1, p_{12} = ah^*(0), p_{13} = bh^*(0), p_{20} = g^*(\lambda), p_{24} = 1 - g^*(\lambda),$

$p_{32} = m^*(0), p_{42} = g^*(0) \cdots (3)$

It can be verified that

$p_{01} = 1; p_{12} + p_{13} = 1; p_{12} + p_{12.3} = 1; p_{20} + p_{24} = 1; p_{20} + p_{22.4} = 1; p_{32} = 1;$

$p_{42} = 1 \qquad \ldots (4)$

The mean sojourn times (μ_i) in the state S_i are

For Model 1

$$\mu_0 = \int_0^\infty P(T > t)dt = m_{01} + m_{02} = \frac{1}{\lambda}, \mu_1 = m_{10} + m_{13} = \frac{1 - g^*(\lambda)}{\lambda},$$

$\mu_2 = m_{21} = -m^{*'}(0),, \mu_3 = m_{31} = -g^{*'}(0),$

$$\mu_1' = m_{10} + m_{11.3} = (1 - g^*(\lambda))\left(\frac{1}{\lambda} - g^{*'}(0)\right) \qquad \cdots (5)$$

For Model 2

$$\mu_0 = \int_0^\infty P(T > t)dt = m_{01} = \frac{1}{\lambda}, \mu_1 = m_{12} + m_{13} = -h^{*'}(0),$$

$\mu_2 = m_{20} + m_{24} = \dfrac{1-g^*(\lambda)}{\lambda}$,

$\mu_1' = m_{12} + m_{13} = -a h^{*'}(0) - b h^{*'}(0) m^*(0) - b m^{*'}(0) h^*(0)$

$\mu_2' = m_{20} + m_{22.4} = \dfrac{(1-g^*(\lambda))(\frac{1}{\lambda} - g^{*'}(0))}{\lambda}$ $\cdots (6)$

Mean Time to System Failure

The regenerative un-failed states to which the system can transit before entering any failed state are i=0,1,2 (model1) ;k_1,k_2=Nil i=0 (model2)

the mean time to system failure (MTSF) is given by

$$\text{MTSF} = \left[\sum_{i,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r(sff)} i)\right\}.\mu_i}{\prod_{k1\neq\xi}\{1 - V(\overline{k1,k1})\}}\right\}\right] \div \left[1 - \sum_{s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r(sff)} \xi)\right\}}{\prod_{k2\neq\xi}\{1 - V(\overline{k2,k2})\}}\right\}\right]$$

=(0,1,0)μ_0+(0,1)μ_1÷ 1-(0,1,0)(For Model 1)

=μ_0+$p_{01}\mu_1$÷ 1-$p_{01}p_{10}$

=N_1÷D_1

MTSF=(0,0)μ_0(for model2)

=μ_0

=N_2÷D_2

For Model1

$$MTSF = (0,1,0)\mu_0 + (0,1)\,\mu_1 \;\div 1 - (0,1,0))$$

$$= \mu_0 + p_{01}\mu_1 \;\div 1 - p_{01}p_{10}$$

$$= \frac{N_1}{D_1}$$

for Model 2

$$MTSF = \{(0,1,2,0) + (0,1,3,2,0\})\,\mu_0 \div (1 - 0)$$

$$= \mu_0 = \frac{N_2}{D_2}$$

Steady State Availability

The regenerative state at which system is available are j=0,1,(for model1)and j=0,2(for model2)

$$A_\xi = \left[\sum_{j,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} j)\right\} f_j.\mu_j}{\prod_{k1\neq\xi}\{1 - V(k1,k1)\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} i)\right\}.\mu_i^1}{\prod_{k2\neq\xi}\{1 - V(k2,k2)\}}\right\}\right]$$

$$A_1 = (1,0)f_0\mu_0 + \{(1,0,1) + (1,0,2,1) + (1,3,1)\}f_1\mu_1 \div [(1,0)\mu_0' + \{(1,0,1) + (1,0,2,1) + (1,3,1)\}\mu_1' + (1,0,2)\mu_2'$$

$$A_2 = (2,0)f_0\mu_0 + \{(2,0,1,2) + (2,0,1,3,2)\}f_2\mu_2 \div [(2,0)\mu_0' + \{(2,0,1)\}\mu_1' + \{(2,1,0,2) + (2,0,1,3,2)\}\mu_2'$$

$$A_i = N_{i0} \div D_{i0,} i = 1,2$$

$N_{10} = p_{10}\mu_0 + \mu_1, D_{10} = p_{10}\mu_0 + \mu_1' + p_{02}p_{10}\mu_2$ (For Model1)

$N_{20} = p_{20}\mu_0 + \mu_2, D_{20} = p_{20}\,\mu_0 + p_{20}\mu_1' + \mu_2'$ (For Model2)

Busy Period of the Server by Inspection /Maintenance/repair

The regenerative state where the server is busy while doing maintenance**/**inspection/repairare j=1,2(for model1)j=1,2,3(for model2)

$$B_\xi = \left[\sum\nolimits_{j,s_r} \left\{ \frac{\left\{pr(\xi \overset{s_r}{\rightarrow} j)\right\}\eta_j}{\prod_{k1\neq\xi}\{1 - V(k1,k1)\}} \right\}\right] \div \left[\sum\nolimits_{i,s_r} \left\{ \frac{\left\{pr(\xi \overset{s_r}{\rightarrow} i)\right\}.\mu_i^1}{\prod_{k2\neq\xi}\{1 - V(k2,k2)\}} \right\}\right]$$

$B_1 = (1,0,1)\eta_1 + (1,0,2)\eta_2 \div D_{10}$

$B_2 = (2,0,1)\eta_1 + \{(2,0,1,2) + (2,0,1,3,2)\}\eta_2 \div D_{20}$

$\mathrm{B_i} = \mathrm{N_{i2}} \div \mathrm{D_{i0,}} \mathrm{i} = 1,2$

$N_{12} = p_{10}p_{01}\eta_1 + p_{10}p_{02}\eta_2$

$N_{22} = p_{20}\eta_1 + \eta_2$

D_{10}, D_{20} are already defined.

Expected Number of Visits of the Server

The regenerative state where the server visits(afresh) for the maintenance/inspection/repairsare j=1(for model1)j=2(for model2)

$$V_\xi = \left[\sum\nolimits_{j,s_r} \left\{ \frac{\left\{pr(\xi \overset{s_r}{\rightarrow} j)\right\}}{\prod_{k1\neq\xi}\{1 - V(k1,k1)\}} \right\}\right] \div \left[\sum\nolimits_{i,s_r} \left\{ \frac{\left\{pr(\xi \overset{s_r}{\rightarrow} i)\right\}.\mu_i^1}{\prod_{k2\neq\xi}\{1 - V(k2,k2)\}} \right\}\right]$$

$V_i = N_{i2} \div D_{i0} \quad i = 1,2$

Where

$N_{12} = p_{10}$

$N_{22} = p_{20}$

D_{10}, D_{20} are already defined.

Profit Analysis

$\mathrm{P_i} = \mathrm{K_0 A_i} - \mathrm{K_1 B_i^r} - \mathrm{K_2 V_i} \qquad (i = 1,2)$

Where

K_0=Revenue per unit up-time of the system.

K_1=Cost per unit time for which server is busy due to maintenance/inspection/repair.

K_2= Cost per unit time visit of the server.

Particular Case

For the particular case $g(t) = \alpha e^{-\alpha t},\ h(t) = \beta e^{-\beta t},\ m(t) = \gamma e^{-\gamma t}$

The following results are obtained:

For Model 1

$N_1 = \frac{\alpha + \lambda + a}{\lambda(\alpha + \lambda)} D_1 = \frac{(\alpha + \lambda) - a\alpha}{(\alpha + \lambda)} N_{10} = \frac{1}{\lambda}$

$N_{11} = \frac{\alpha}{\alpha + \lambda}.\frac{b}{\gamma} N_{12} = \frac{1}{\alpha} N_{13} = \frac{\alpha}{\alpha + \lambda}$

$D_{10} = \frac{\alpha}{\alpha + \lambda}\left[\frac{1}{\lambda} + \frac{b}{\gamma}\right] + \frac{1}{\alpha}$

For Model 2

$N_2 = 1, \qquad D_2 = \lambda, \qquad N_{20} = \frac{1}{\lambda}$

$N_{21} = \frac{\alpha}{\alpha + \lambda}\left[\frac{1}{\beta} + \frac{b\beta}{\gamma(\gamma + \beta)}\right], \qquad N_{22} = \frac{1}{\alpha}, \qquad N_{23} = \frac{\alpha}{\alpha + \lambda}$

$D_{20} = \frac{\alpha}{\alpha + \lambda}\left[\frac{1}{\lambda} + \frac{1}{\beta} + \frac{b}{\gamma}\right] + \frac{1}{\alpha}$

Comparative Analysis

There is a sudden decline in mean time to system failure (MTSF) with the increase of failure rate (λ) for other fixed parametric values as shown in figures 3 and 4. Figures 5,6 and 7,8 indicate that profit of the system models go on decreasing as failure rate (λ) increases. However, system becomes more profitable when repair and maintenance rates increase provided unit has more chances of deterioration in standby mode. Also, it is interesting to note that system model 1 has more values of MTSF and profit. The profit comparison of the models is shown in figure 9. Hence, we can say that maintenance policy 2nd should not be adopted in case of a two-unit cold standby system.

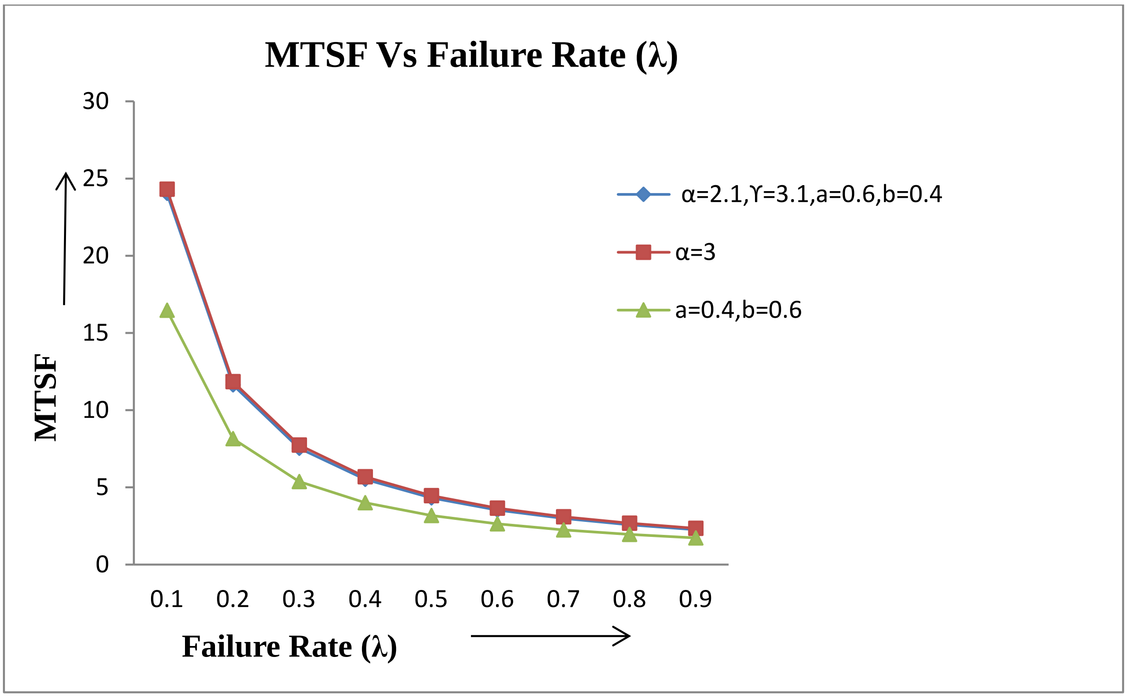


Fig. 3: Model1

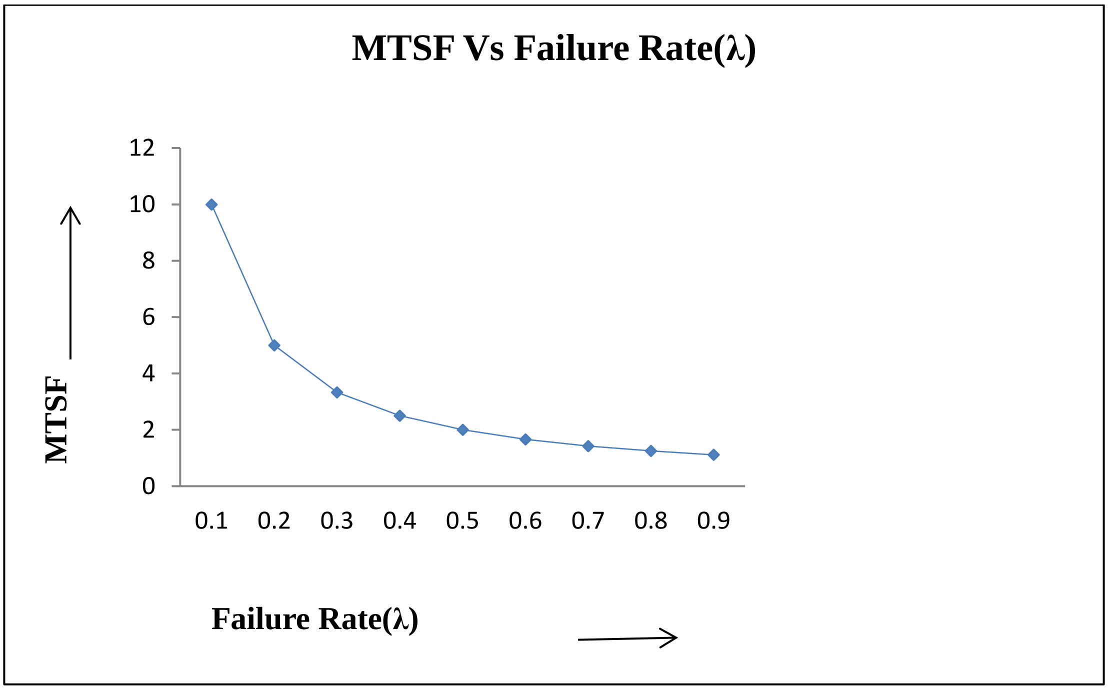
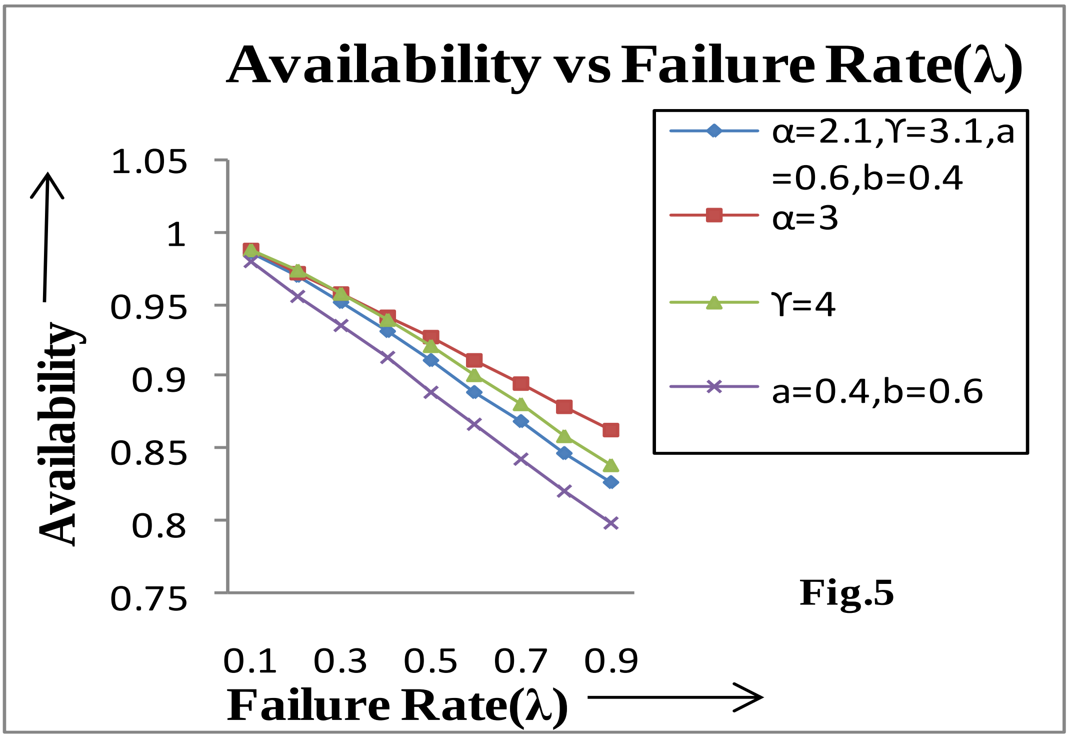
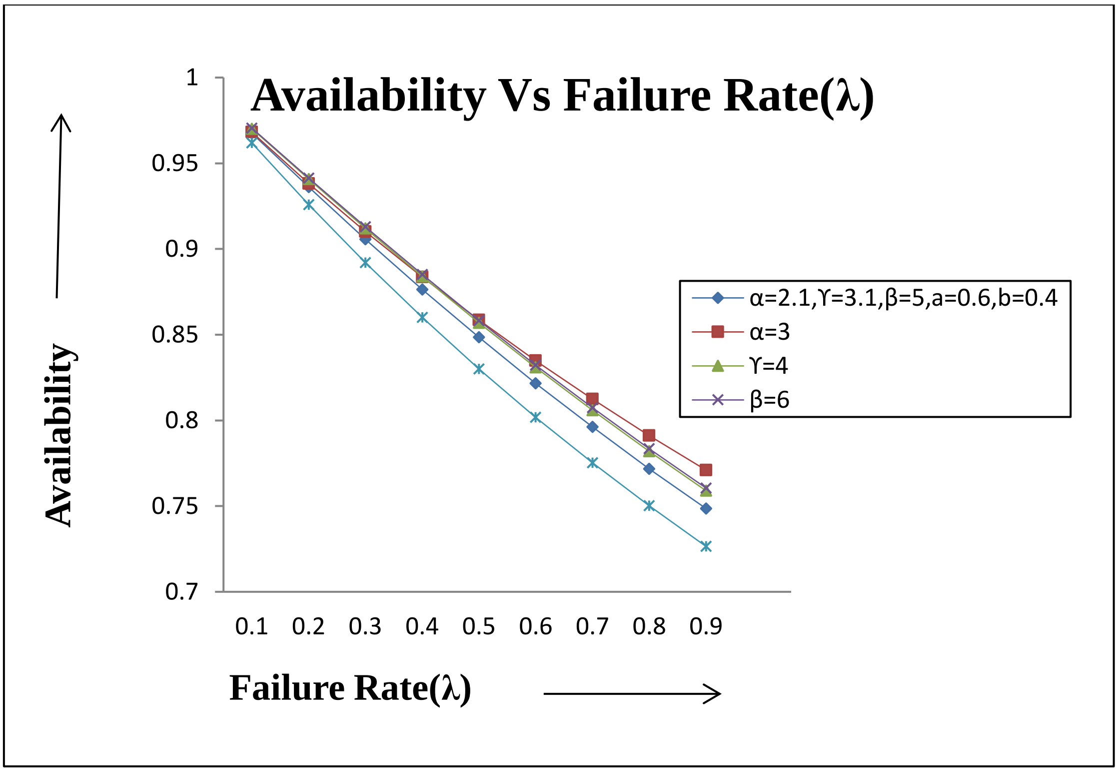


Fig.4: Model2



Availability vs Failure Rate(λ)
α=2.1,ϒ=3.1,a =0.6,b=0.4
α=3
ϒ=4
a=0.4,b=0.6
1.05
1
0.95
0.9
0.85
0.8
0.75
0.1 0.3 0.5 0.7 0.9
Availability
Failure Rate(λ)

Fig.5



Availability Vs Failure Rate(λ)
α=2.1,ϒ=3.1,β=5,a=0.6,b=0.4
α=3
ϒ=4
β=6
1
0.95
0.9
0.85
0.8
0.75
0.7
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Availability
Failure Rate(λ)

Fig.6

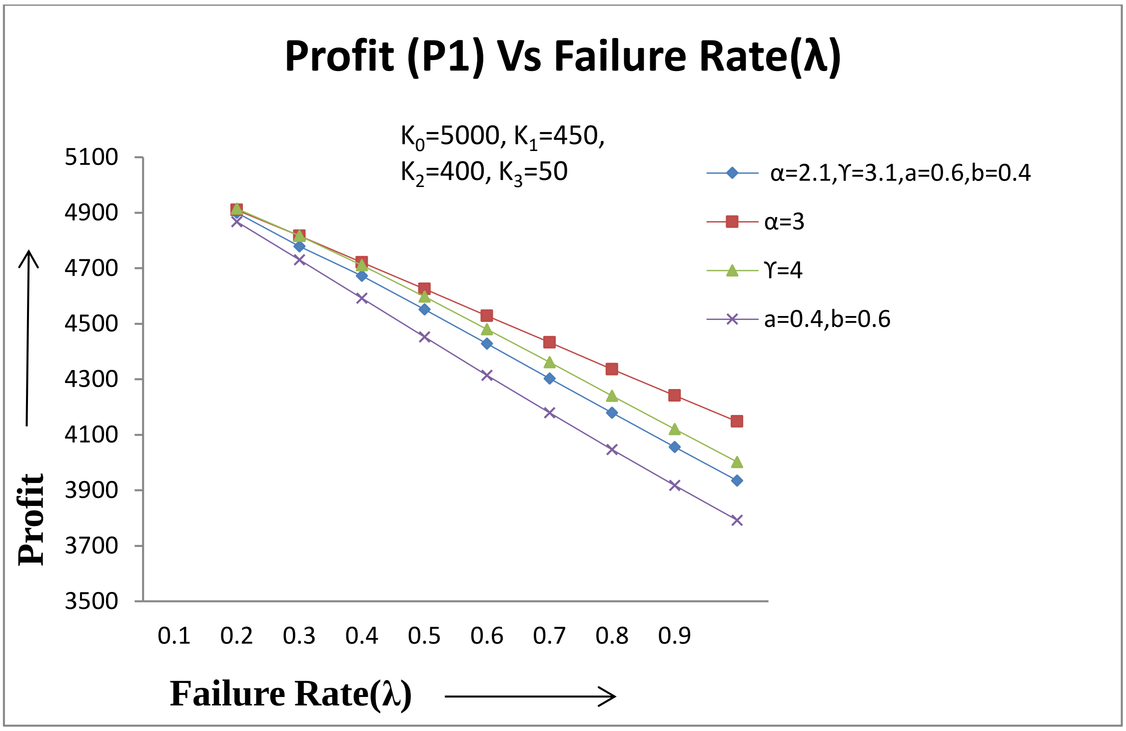


Fig.7: Model1

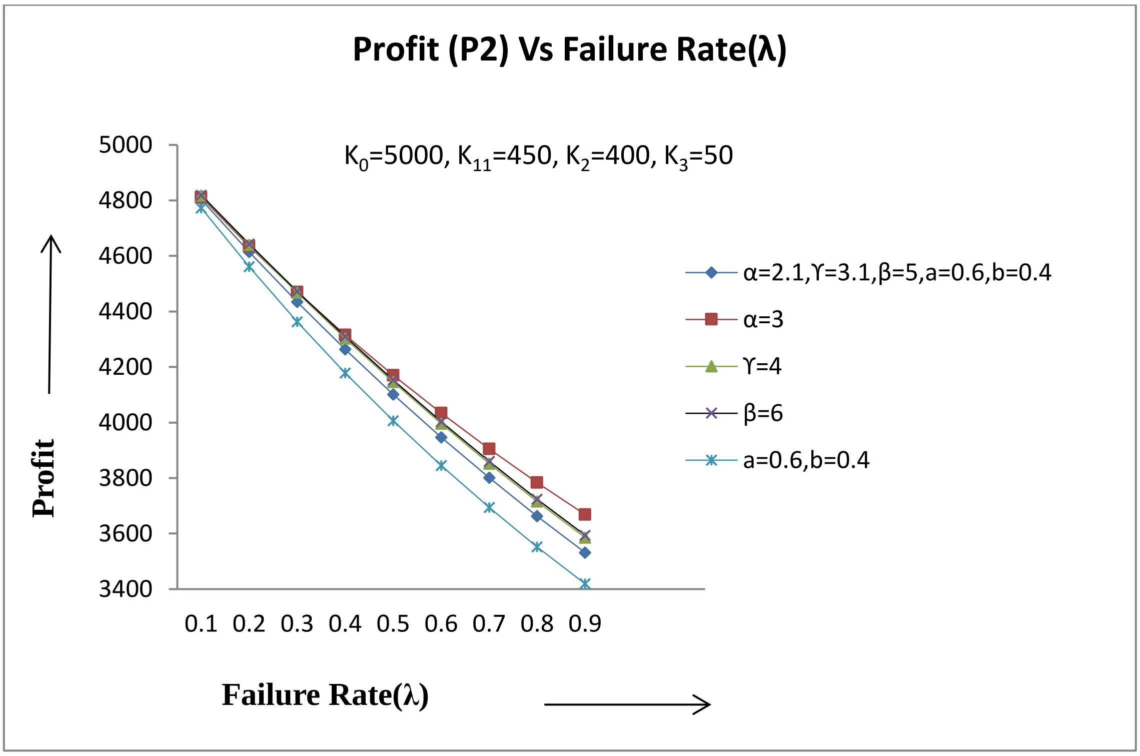


Fig.8: Model 2

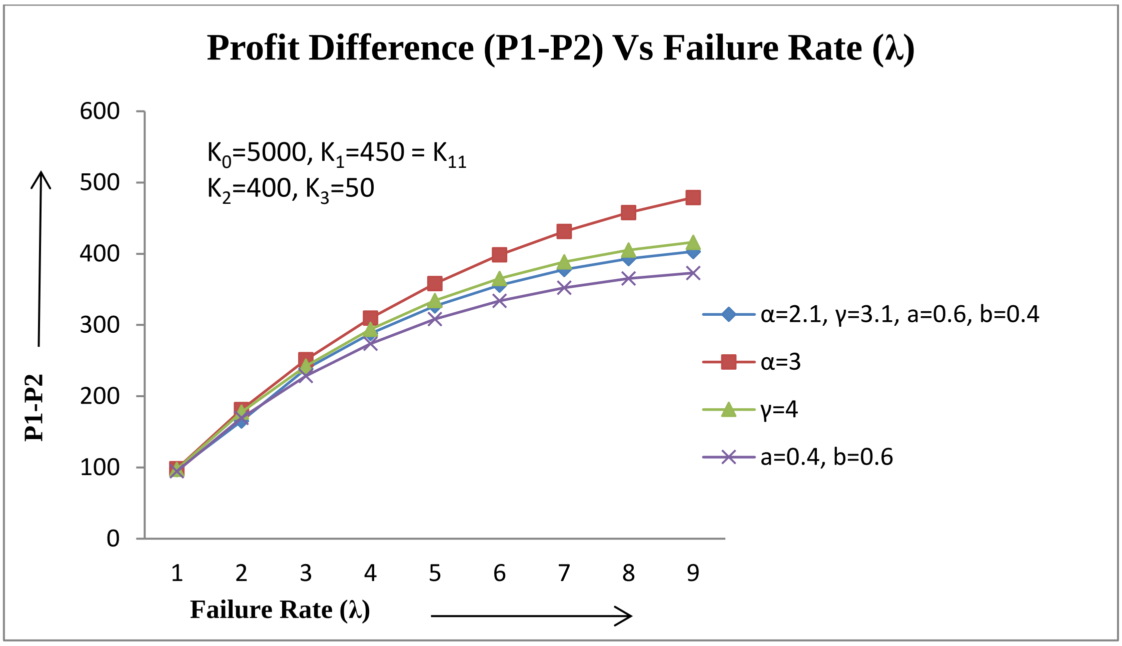


Fig.9: Profit Difference

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