International Journal of Education, Modern Management, Applied Science & Social Science (IJEMMASSS)
ISSN : 2581-9925, Impact Factor: 6.340, Volume 03, No. 03(III), July - September, 2021, pp. 255-261

MATHEMATICAL AND GRAPHICAL STUDY OF MEAN QUEUE LENGTH OF BISERIAL FEEDBACK QUEUING SYSTEM HAVING FACILITY OF THREE PARALLEL SERVERS CONNECTED WITH A COMMON FOURTH SERVER

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ABSTRACT

The present paper deals with the mathematical and graphical study of mean queue length of a queueing system of a developed queuing model in which three servers are connected in parallel in tri-cum biserial way with a common server. The arrival and service pattern are assumed to follow the Poisson process. Variation in Mean queue length of the system have been studied mathematically and graphically by changing the values of different queueing characteristics keeping the remaining variables constant.

Keywords: *Four Servers, Mathematical and Graphical Study, Mean Queue Length, Queueing Characteristics.*

Introduction

Feedback queues are those queues which are formed due to rejoining of the queuing system by the unsatisfied customers who have already availed the service. In real life feedback queues are observed generally in manufacturing concerns, offices, in hospitals etc. A large number of researchers have done a lot of work on queuing models considering various aspects such as bulk service, impatient customers, cyclic queues, batch arrivals, reneging, blocking. Study on queuing models considering the concept of feedback has also been done by a large number of researchers. Jacksons (1954) was the first who investigated the various characteristics of a queue system encompassing phase type service. Maggu (1970) emphasized on phase type service queues with two servers in biseries to study the queue model. Singh *et al (*2005*)* used parallel biserial queues to examine the transient behavior of a queuing network.

The state characteristics of a queue model having two subsystems with bi-serial channels connected with a common channel has been investigated by Kumar et al (2006). Gupta et al (2007) presented an extensive parametric study to explore the queuing model consist of biserial and parallel channels associated with a common server. Gupta et al (2012) presented a detailed investigation on the linkage of a flowshop scheduling model with a parallel biserial queue network. This work has been further explored by Seema et al to optimize total flow time, waiting time and service time.

Kumar and Taneja (2019) also studied the many feedback queuing models. They studied the feedback queuing models with the repitititon of service once and more than once but finite times. Aggarwl and singh (2018) deal with the investigation of various characteristics queue length, variance, average waiting time and probabilities of a developed queuing model in which three servers are connected in parallel in tri-cum biserial way with a common server. They did not find the queue length at each of the server.

Santosh Kumari (2021) found the expected queue length at each of the four server and the whole biserial feedback queuing system having facility that the three parallel servers connected with a common fourth server by using the value of generating function of the model developed by Aggarwal and Singh (2018).

In the present paper, I have discussed the variation in mean queue length of the system with the help of the tables and graphs by putting different possible values of different variables in the formula of expected queue length of the whole queueing system solved by me i.e. Santosh Kumari (2021). Arbitrary different possible values of queueing characteristics have been put in the formula of expected queue length derived by me by keeping the other variables constant.

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Notations

In the present work, a queue model consist of three servers (S_a, S_b and S_c) are connected in parallel in tri-cum biserial way which are further connected to a common server S_d. Let Lq_1, Lq_2, Lq_3 and Lq_4 be the queues associated with servers S_a, S_b, S_c and S_d respectively. Let n_a, n_b, and n_c be the number of customers at any time t coming at S_a, S_b and S_c from outside with mean arrival rate λ_a, λ_b, and λ_c respectively. Also, n_d be the no. of customers at server S_d at any time t.

λ_a : Mean Arrival rate at 1st server (S_a)

λ_b : Mean Arrival rate at 2nd server (S_b)

λ_c : Mean Arrival rate at 3rd server (S_c)

μ_a: service rate of 1st server (S_a)

μ_b: service rate of 2nd server (S_b)

μ_c: service rate of 3rd server (S_c)

μ_d: service rate of 4th server (S_d)

P_{ij}: the probability of customer going from i^{th} to j^{th} server.

P_i: the probability of customer going from i^{th} to outside the whole queueing system

L_q : denote the Mean Queue Length of the system.

The Expected queue length of the whole system by Santosh Kumari (2021) is given by:

$$L_q = \mu_a\left\{\frac{\mu_a f_1 - \mu_b P_{ba} f_2 - \mu_c P_{ca} f_3}{(-\lambda_a + \mu_a - \mu_b P_{ba} - \mu_c P_{ca})^2} - \frac{f_1}{(-\lambda_a + \mu_a - \mu_b P_{ba} - \mu_c P_{ca})}\right\}$$
$$+\mu_b\left\{\frac{-\mu_a f_1 P_{ab} + \mu_b f_2 - \mu_c f_3 P_{cb}}{(-\lambda_b - \mu_a P_{ab} + \mu_b - \mu_c P_{cb})^2} - \frac{f_2}{(-\lambda_b - \mu_a P_{ab} + \mu_b - \mu_c P_{cb})}\right\}$$
$$+\mu_c\left\{\frac{-\mu_a P_{ac} f_1 - \mu_b P_{bc} f_2 + \mu_c f_3}{(-\lambda_c - \mu_a P_{ac} - \mu_b P_{bc} + \mu_c)^2} - \frac{f_3}{(-\lambda_c - \mu_a P_{ac} - \mu_b P_{bc} + \mu_c)}\right\}+$$
$$\mu_d\left\{\frac{-\mu_a P_{ad} f_1 - \mu_b P_{bd} f_2 - \mu_c f_3 P_{cd} + \mu_d f_4}{(-\mu_a P_{ad} - \mu_b P_{bd} - \mu_c P_{cd} + \mu_d)^2} - \frac{f_4}{(-\mu_a P_{ad} - \mu_b P_{bd} - \mu_c P_{cd} + \mu_d)}\right\}$$

Numerical Results and Discussion

- Behaviour of mean queue length (L_q) of the system with respect to mean arrival rate of customers at first server (λa) for different values of mean arrival rate of customers at second server (λ_b) is depicted in Table 1 and in Fig. 1 keeping the values of other parameters as fixed.

Table 1

λ_c=15, μ_a=11, μ_b=12, μ_c=13, μ_d=30, p_{ab}=0.1, p_{ac}=0.3, p_{ad}=0.6, p_{ba}=0.2, p_{bc}=0.4, p_{bd}=0.4, p_{ca}=0.3, p_{cb}=0.5, p_{cd}=0.2			
	λ_b=12	λ_b=13	λ_b=14
λ_a	L_q	L_q	L_q
10	52.50113	53.53577	54.55742
11	58.63082	59.64303	60.64256
12	64.63822	65.63729	66.62402
13	70.56209	71.553	72.53187
14	76.4266	77.41222	78.38612
15	82.24752	83.2296	84.20029
16	88.03553	89.0152	89.98381
17	93.79808	94.77611	95.74339
18	99.54056	100.5175	101.4839
19	105.2669	106.2431	107.2091

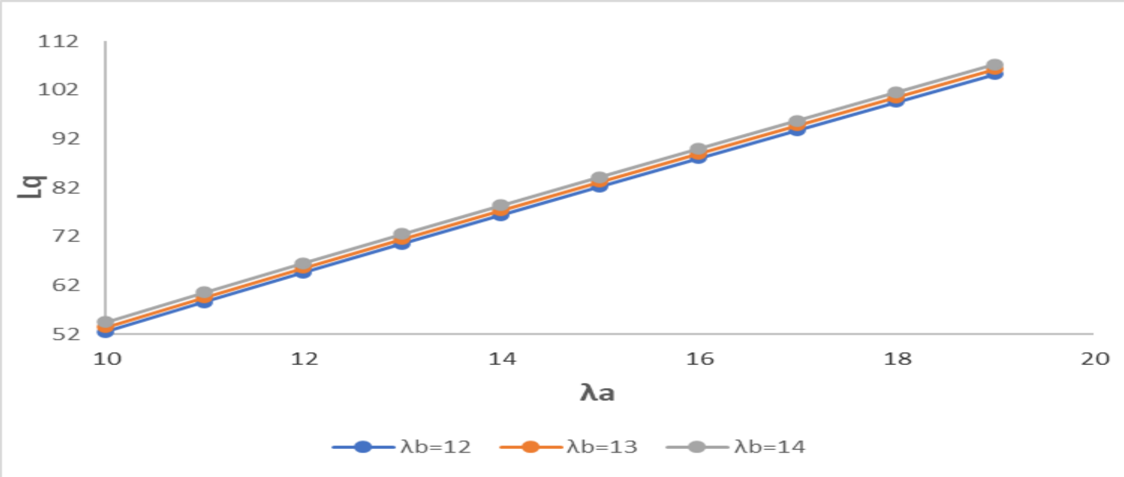

Fig. 1

Following can be interpreted from **Table 1** and **Fig. 1.**

- Mean queue length of the system L_q get increased with the increase in λ_a.
- Mean queue length of the system L_q increases with the increase in λ_b.

- Behaviour of mean queue length (L_q) of the system with respect to mean arrival rate of customers at first server (λa) for different values of mean service rate of first server (μ_a) is depicted in Table 2 and in Fig. 2 keeping the values of other parameters as fixed.

Table 2

λ_b=12, λc=15, μ_b=12, μ_c=13, μ_d=30, p_{ab}=0.1, p_{ac}=0.3, p_{ad}=0.6, p_{ba}=0.2, p_{bc}=0.4, p_{bd}=0.4, p_{ca}=0.3, p_{cb}=0.5, p_{cd}=0.2			
	μ_a=11	μ_a =12	μ_a =15
λ_a	L_q	L_q	L_q
10	52.50113	53.01611	41.54914
11	58.63082	59.92439	58.78593
12	64.63822	66.59928	70.42974
13	70.56209	73.1267	80.33576
14	76.4266	79.5554	89.47832
15	82.24752	85.91491	98.21809
16	88.03553	92.2241	106.7195
17	93.79808	98.49559	115.0682
18	99.54056	104.7381	123.3131
19	105.2669	110.9578	131.4845

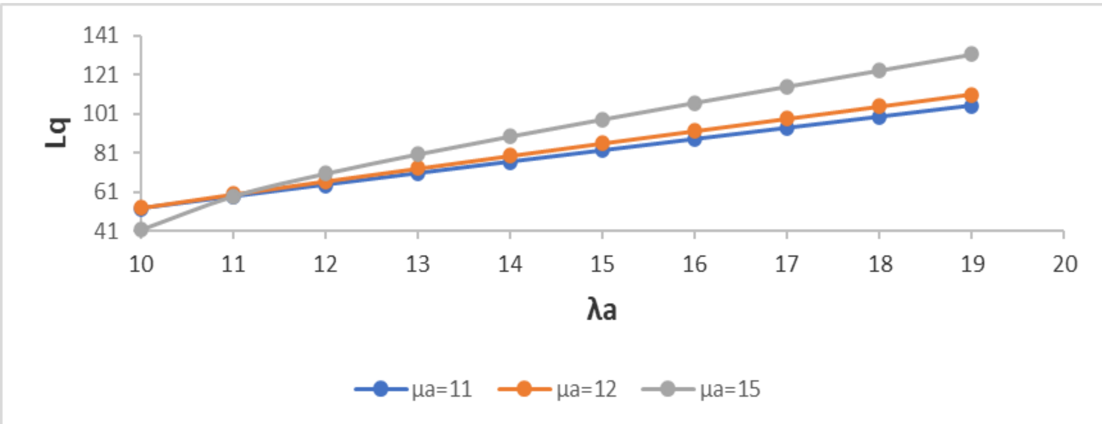

Fig. 2

Following can be interpreted from Table 2 and Fig. 2.

- Mean queue length of the system L_q get increased with the increase in λ_a.
- Mean queue length of the system L_q initially increases with the increase in μa but decreases for greater than μa=12.and increases continuously with the increase in service rate μa after λ_a =11.

- Behaviour of mean queue length (L_q) of the system with respect to mean arrival rate of customers at first server (λa) for different values of mean service rate of fourth server (μ_d) is depicted in Table 3 and in Fig. 3 keeping the values of other parameters as fixed.

Table 3

λ_b=12, λc=15 , μ_a=11, μ_b=12, μ_c=13, p_{ab}=0.1, p_{ac}=0.3, p_{ad}=0.6,p_{ba}=0.2, p_{bc}=0.4, p_{bd}=0.4,p_{ca}=0.3, p_{cb}=0.5, p_{cd}=0.2			
	μ_d=11	μ_d=12	μ_d=15
λa	L_q	L_q	L_q
10	463.9251	1147.129	5877.708
11	492.0317	1218.745	6288.538
12	520.0161	1290.239	6699.245
13	547.917	1361.65	7109.869
14	575.7585	1433.001	7520.433
15	603.5565	1504.309	7930.954
16	631.3215	1575.584	8341.442
17	659.061	1646.833	8751.905
18	686.7805	1718.062	9162.347
19	714.4839	1789.275	9572.774

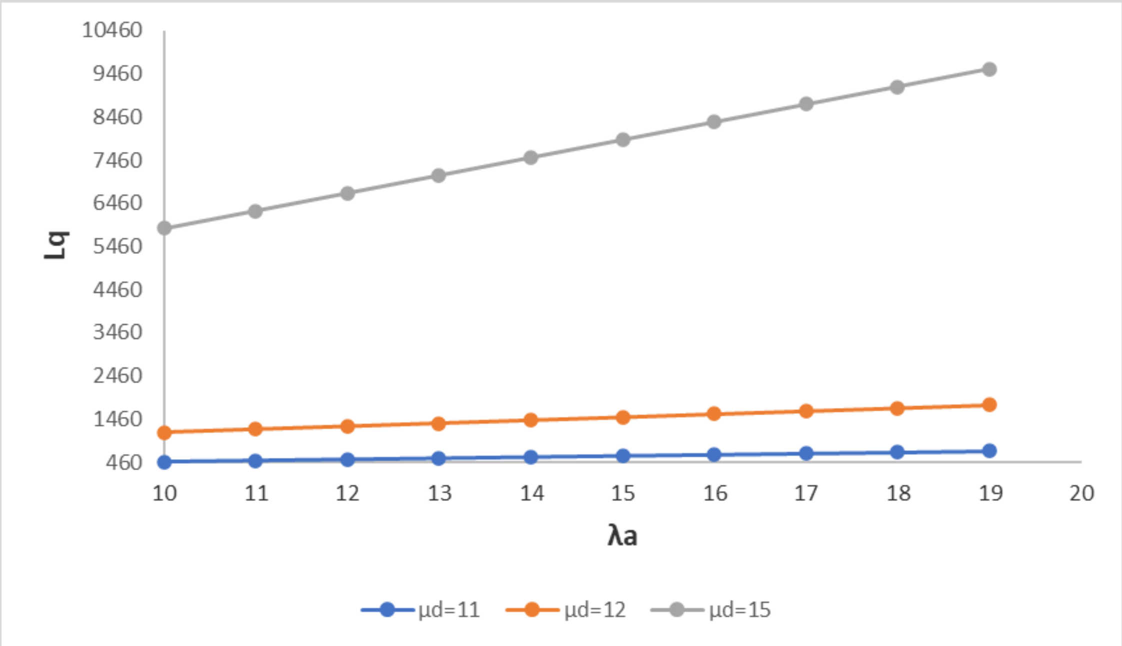

Fig.3

From Table 3 and Fig. 3 we interpret that Mean queue length of the system L_q increases with the increase in mean arrival rate of first server (λ_a) as well as increase in service rate of fourth server (μ_d).

- Behaviour of mean queue length (L_q) of the system with respect to mean arrival rate of customers at second server (λ_b) for different values of mean service rate of second server (μ_b) is depicted in Table 4 and in Fig. 4 keeping the values of other parameters as fixed.

Table 4

λ_b	μ_b=10	μ_b =12	μ_b =13
λa=10, λc=15 , μ_a=11, μ_b=12, μ_c=13, p_{ab}=0.1, p_{ac}=0.3, p_{ad}=0.6, p_{ba}=0.2, p_{bc}=0.4, p_{bd}=0.4, p_{ca}=0.3, p_{cb}=0.5, p_{cd}=0.2			
	L_q	L_q	L_q
10	812.1436	444.0722	348.2563
11	829.8108	453.9995	356.0915
12	847.4623	463.9251	363.9753
13	865.1001	473.8393	371.8645
14	882.7265	483.7406	379.7459
15	900.3433	493.6295	387.616
16	917.952	503.5076	395.4748
17	935.554	513.3763	403.323
18	953.1502	523.2369	411.1621
19	970.7416	533.0906	418.9931

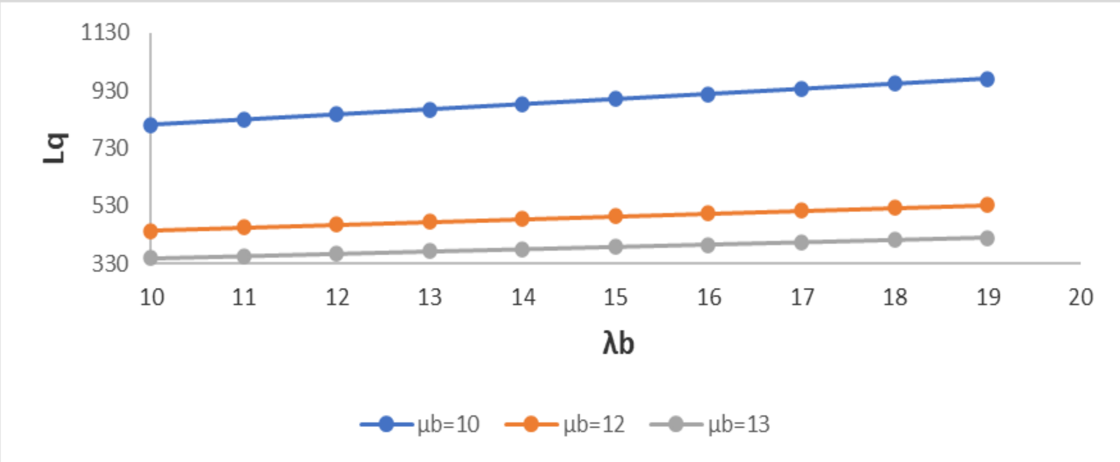

Fig. 4

Following can be interpreted from Table 4 and Fig. 4.

- Mean queue length of the system L_q increases with the increase in λ_a.
- Mean queue length of the system L_q decreases with the increase in service rate of second server (μ_b).

- Behaviour of mean queue length (L_q) of the system with respect to probability of customers from first server to second server (p_{ab}) for different values of mean arrival rate of second server (λ_b) is depicted in Table 5 and in Fig. 5 keeping the values of other parameters as fixed.

Table 5

p_{ab}	λ_b=10	λ_b =15	λ_b =18
λa=10, λc=15 , μ_a=11, μ_b=12, μ_c=13, μ_d=11, p_{ba}=0.2, p_{bc}=0.4, p_{bd}=0.4, p_{ca}=0.3, p_{cb}=0.5, p_{cd}=0.2			
	L_q	L_q	L_q
0.25	959.4051	1020.595	1057.924
0.3	1004.375	1069.014	1108.307
0.35	1054.089	1122.733	1164.334
0.4	1109.211	1182.492	1226.794
0.43	1173.796	1252.331	1299.74
0.5	1238.868	1323.73	1374.855
0.53	1318.257	1409.965	1465.168
0.58	1403.851	1503.779	1563.875
0.63	1500.247	1609.748	1675.556
0.69	1608.196	1729.315	1802.068

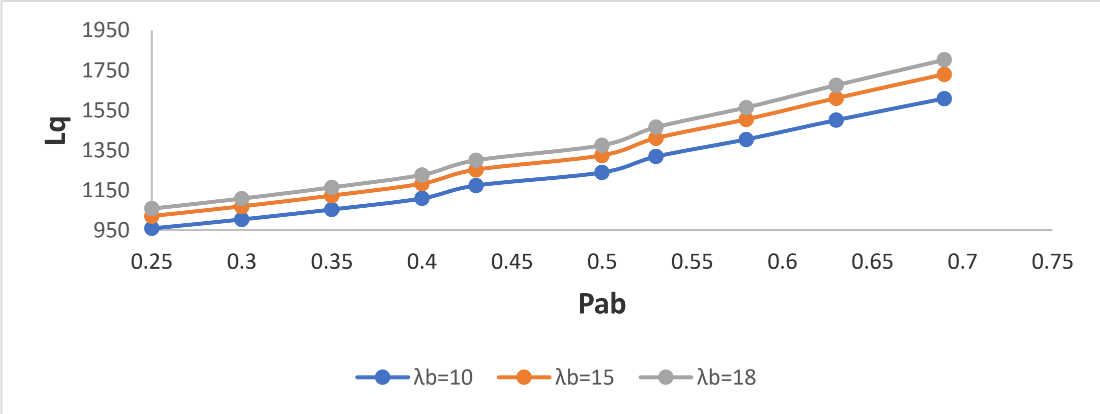

Fig. 5

The Following can be interpreted from Table 5 and Fig. 5

Mean queue length of the system L_q increases with the increase in p_{ab} as well as increase in mean arrival rate of second server **(λ_b).**

- Behaviour of mean queue length (L_q) of the system with respect to probability of customers visiting from third server to fourth server (p_{cd}) for different values of mean arrival rate of second server (μ_d) is depicted in Table 6 and in Fig. 6 keeping the values of other parameters as fixed.

Table 6

$\lambda a=10$, , $\lambda_b=12$, $\lambda c=15$, $\mu_a=11$, $\mu_b=12$, $\mu_c=13$, $p_{ab}=0.3$, $p_{ac}=0.2$, $p_{ad}=0.5$, $p_{ba}=0.2$, $p_{bc}=0.35$, $p_{bd}=0.45$			
p_{cd}	$\mu_d=2$	$\mu_d=5$	$\mu_d=7$
	L_q	L_q	L_q
0.15	4.037182	23.91505	63.16923
0.155	5.291838	25.6459	65.75837
0.16	6.472284	27.2593	68.12119
0.165	7.62504	28.80128	70.30615
0.17	8.42699	29.78561	71.40017
0.175	10.01836	31.83828	74.31519
0.18	10.9128	32.84408	75.27864
0.185	12.37047	34.51575	77.20326
0.19	14.11195	36.4195	79.25233
0.195	16.80026	39.26511	82.27129

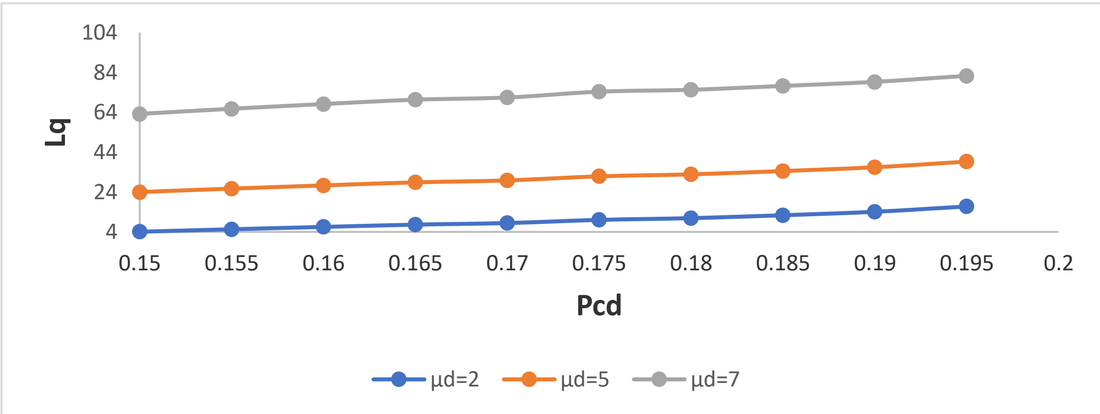

Fig.6

Following can be interpreted from Table 6 and Fig. 6

Mean queue length of the system L_q increases with the increase in probability p_{cd}. as well as the increase in mean service rate of fourth server **(μ_d).**

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