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MODEL ORDER REDUCTION USING EIGEN SPECTRUM ANALYSIS AND ROUTH ARRAY METHOD

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ABSTRACT

The author suggests a hybrid technique to reduce the high order dynamic systems. The suggested approach uses the Routh array method to obtain the coefficients of the numerator and the Eigen Spectrum Analysis of the original system to generate the denominator polynomial of the lower order system, which is preserved in the reduced model. This technique is easy and simple and generates stable reduced models if the original high- order system is stable. The suggested technique is illustrated with the help of the numerical example taken from the literature.

Keywords: *Model Order Reduction, Eigen Spectrum, Routh Array, Stability, Transfer Function.*

Introduction

In many engineering applications, particularly in control system design, where an engineer must govern a physical system for which an analytic model is represented as a high order linear system, the approximation of linear systems plays a crucial role. A rather complicated and high order system is not only laborious but also expensive to execute online in many real-world scenarios. Therefore, it is preferable that a low order system take the place of a high order system while maintaining the majority of the former system's qualitative characteristics.

The several order reduction techniques for linear dynamic systems in the frequency domain are available the literature [1-5]. Further, some methods have also been suggested by combining the features of two different methods [6-7]. The Pade approximation method was originally introduced by Pade [8]. This method is computationally simple and fits initial time moments and matches the steady state values. The disadvantage of this Method is that the reduced model may be unstable even though the original system is stable. Sumit Mondal [9] utilizing the basic characteristics of original system and pade approximation to get reduced order system. Authors proposed that, the denominator polynomial of reduced model is obtained using basic characteristics such as undamped natural frequency ofoscillations (ω_n), damping ratio(£),settling time(T_s), peak overshoot(M) & peak time(t_p) while the coefficient of numerator of reduced order system is obtained by using Routh array can be [24]. This method is computationally simple and is applicable to stable systems.

In the next section, the algorithm is described in detail with the help of numerical example.

Statement of the Problem

Let the transfer function of high order original systemof the order 'n' be

$$G(s) = \frac{a_0 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + \ldots\ldots + a_{n-1}}{s^n + b_1 s^{n-1} + b_2 s^{n-2} + b_3 s^{n-3} + \ldots\ldots + b_n} \qquad (1)$$

Where a_i; $0 \le i \le n-1$ and b_i; $0 \le i \le n$ known scalarconstants.

$$R_k(s) = \frac{c_0 + c_1 s + c_2 s^2 + \cdots + c_{k-1} s^{k-1}}{d_0 + d_1 s + d_2 s^2 + \cdots d_k s^k} \qquad (2)$$

Let the transfer function of the reduced model of theorder 'k' be

c_j ; $0 \le j \le k-1$ and d_j ; $0 \le j \le k$ are unknown scalar constants.

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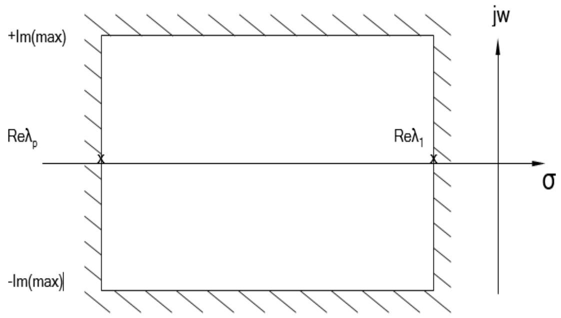
The aim of this paper is to realize the k^{th} order reduced model in the form of (2) from the original system (1) such that it retains the important features of the original high -order system.



(a) eigen spectrum zone (ESZ) of HOS

Reduction Method

The reduction procedure for getting the k^{th}-order reduced models consists of the following two steps:

- **Step-1:** Determination of the denominator polynomial for the k^{th}-order reduced model using Eigen Spectrum Analysis of original system by the following procedure:

Step 1: First, as illustrated in Fig. 1, adjust the Eigen spectrum zone (ESZ) of the HOS. The ESZ is formed by the two lines that cut through the nearest ($Re\lambda_1$) and farthest ($Re\lambda p$) real poles when cut by two lines that pass through the farthest imaginary pole pairs (±Im(max)). This is the case if poles -λi (i = 1,n) are positioned at – (Reλi ± Imλi) (i = 1,p) within the ESZ.

Step 2: Measurement of the HOS stiffness and pole centroid:

The mean of the real parts of the poles is known as the pole centroid, and it may be represented as:

$$\lambda_m = \frac{\sum_{i-1}^{p} R_e \lambda_i}{p} \qquad (3)$$

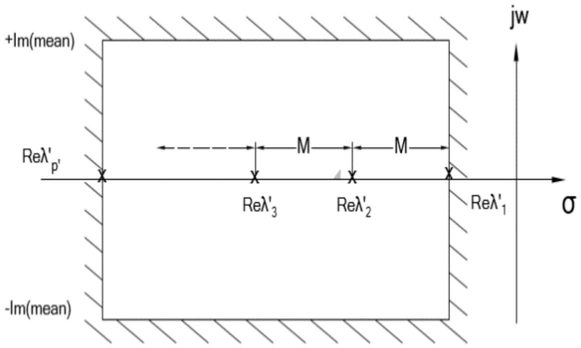
System stiffness is defined as the ratio of the nearest to the farthest pole of a system in terms of real parts only and is put as

$$\lambda_s = \frac{R_e \lambda_1}{R_e \lambda_p} \qquad (4)$$

Step 3: Determination of eigen spectral points of LOS:

If λ'_m and $\lambda'_s$are pole centroid and system stiffness of LOS such that $\lambda'_m = \lambda_m$ and $\lambda'_s = \lambda_s$ then following situation arise:

$$\lambda'_s = \frac{R_e \lambda'_1}{R_e \lambda'_{p'}} = \lambda_s$$



(b) eigen spectrum zone (ESZ) of LOS

$$\lambda'_m = \frac{R_e \lambda'_1 + R_e \lambda'_2 + R_e \ldots\ldots + R_e \lambda'_{p'}}{p'} = \lambda_m \qquad (5)$$

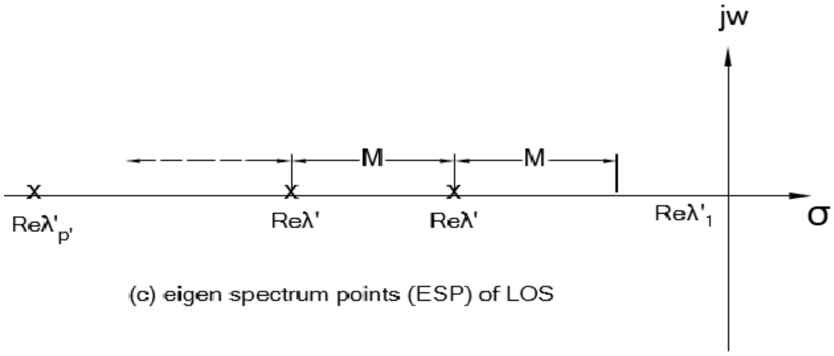


Fig.1: Eigen Spectrum Zones and Points of System

Where λ'_i(i=1,r) are the poles of LOS located at – (Reλ'_i ± Imλ'_i) i=1,p'. Now if,

$$\frac{R_e\lambda'_{p'} - R_e\lambda'_1}{p'-1} = M \,, \qquad (6)$$

i.e. $R_e\lambda'_1$+M=$R_e\lambda'_2$, $R_e\lambda'_2$+M=$R_e\lambda'_3$ and so on till $R_e\lambda'_{p'-1} + M = R_e\lambda'_{p'}$ then Eq. (5) can be put as

$$\lambda_m = \frac{R_e\lambda'_1 + R_e\lambda'_{p'} + - - \mp(R_e\lambda'_1 + (p'-2)M)}{p'}$$

Or $\lambda_m p' = R_e\lambda'_1 + R_e\lambda'_{p'} + (M + 2M + \cdots ... + (p'-2)M)$

Or N= $R_e\lambda'_1$(p'-1) + $R_e\lambda'_{p'}$+ QM (7)

Where N=λ'_m and QM = M+2M+ ……………………+ (p'-2) M.

By putting$R_e\lambda'_1 = \lambda_s R_e\lambda'_{p'}$. Equation (6) and (7) as under:

$R_e\lambda'_{p'} - \lambda_s R_e\lambda'_{p'} = M(p'-1)$ (8)

$R_e\lambda'_{p'}(p'-1) + R_e\lambda'_{p'}$+QM (9)

Eqs. (8) and (9) can be put as

$\lambda'_{p'}(1-\lambda_s) + M(1-p') = 0$

$R_e\lambda'_{p'}[\lambda_s(p'-1)+1] + MQ = N \qquad or$

$$\begin{bmatrix} \lambda_s(p'-1)+1 & Q \\ (1-\lambda_s) & (1-p') \end{bmatrix} \begin{bmatrix} R_e\lambda'_{p'} \\ M \end{bmatrix} = \begin{bmatrix} N \\ 0 \end{bmatrix} \qquad (10)$$

Eq. (10) can be solved for $R_e\lambda'_{p'}$ and M enabling thereby to locate the Eigen spectral points (ESP) as shown in Fig. 1.

Step 2

The Routh array method is based on Routh stability criterion. In this method, none of the algorithm is required. This is a very simple procedure for reducing model for a high- order system. The array is usually written in the following form, from the coefficient of a given polynomial [6].

G(s) = $\frac{a_1 s^{n-1} + a_2 s^{n-2} + a_3 s^{n-3} + ...+ a_n}{s^n + b_1 s^{n-1} + b_2 s^{n-2} + b_3 s^{n-3} + ...+ b_n}$ (11)

Where, n=2, 3, 4…………..

The routh array for the numerator and denominator polynomials of Eq.3. are shown in below in Table I and Table-II.

Table 1: Routh Stability Array for Numerator

a_{11}	a_{12}	a_{13}	a_{14}	–
a_{21}	a_{22}	a_{23}	a_{24}	–
a_{31}	a_{32}	a_{33}	–	
–	–	–	–	
$a_{n,1}$	–			
$a_{n+1,1}$	–			

Table 2: Routh Stability Array for Dinominator

$\boldsymbol{b_{11}}$	$\boldsymbol{b_{12}}$	$\boldsymbol{b_{13}}$	$\boldsymbol{b_{14}}$	–
$\boldsymbol{b_{21}}$	$\boldsymbol{b_{22}}$	$\boldsymbol{b_{23}}$	$\boldsymbol{b_{24}}$	–
$\boldsymbol{b_{31}}$	$\boldsymbol{b_{32}}$	$\boldsymbol{b_{33}}$	–	
–	–	–	–	
$\boldsymbol{b_{n-1,1}}$	–			
$\boldsymbol{b_{n,1}}$	–			

The first, third and fifth, etc. rows of each table consists of odd coefficients and the second, fourth and sixth, etc. rows of each table consist of even coefficients. The first two are generated from the given transfer function and remaining rows are calculated by below equation Eq.4

$C_{ij} = C_{i-2,j+2} - \frac{(C_{i-2,1}\, x C_{i-1,j+1})}{C_{i+,1,1}}$

For i≥3 and i≤$\left[\left(\frac{(n-1+3)}{2}\right)\right]$,

Here "n" refers to the order of the polynomials. So that the first two rows are made available for each table from given polynomials, then the other rows are easily reconstructed by this approach. In paper, this approach is used for finding reduced order numerator of given high-order transfer function.

Method for Comparison

In order to check the accuracy of the proposed method the relative integral square error ISE index in between the transient parts of the reduced models andthe original system is calculated using Matlab / Simulink.

The integral square error ISE is defined as:

$$ISE = \int_0^\infty [y_n(t) - y_k(t)]^2\, dt$$

Numerical Example

The proposed method explains by considering numerical example, taken from the literature. The goodness of the proposed method is measured by calculating the rise time (t_r), settling time (t_s) and maximum overshoot (M_p) and compare with the original system.

Example: Consider a fourth order system from the literature

$$G(s) = \frac{24+24s+7s^2+s^3}{24+50s+35s^2+10s^3+s^4} \quad (12)$$

Step 1: Denominator of reduced order model is determine using following Eigen Spectrum Analysis of original system where $\lambda_1 = -1, \lambda_2 = -2, \quad \lambda_3 = -3, \quad \lambda_4 = -4$

Correcting the HOS's ESZ

It will be a line connecting the closest and furthest poles because all poles are real.

Step 2: Measurement of the HOS stiffness and pole centroid:

$$\lambda_m = \frac{\sum_{i=1}^{10} \lambda_i}{10} = 2.5$$

$$\lambda_s = \frac{\lambda_1}{\lambda_4} = 0.25$$

Step 3: Determination of Eigen spectral points of LOS:

Eq. (10) can be formed as under:

$$\begin{bmatrix} 1.25 & 0 \\ 0.75 & -1 \end{bmatrix} \begin{bmatrix} \lambda'_{2'} \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

$\lambda'_{2'} = 4$

$\lambda'_{1'} = 1$

Therefore $\widetilde{D}_2(s) = s^2 + 5s + 4$

Step 4: Determination of numerator using Routh Array

$N_2(s) = 3.4271s + 3.9984$

$R_2(s) = \frac{N2(s)}{D2(s)} = \frac{20.571s+24}{s^2+5s+4}$

Applying steady state correction to reduced order model.

SSO = $\frac{24}{24}$ = 1

SSR = $\frac{24}{4}$ = 6

K2 = SS0/SSR2 = 1/6 = 0.1666

So, that the finally second order transfer function are shown below in eq.

$R_2(s) = \frac{3.4271s + 3.9984}{s^2 + 5s + 4}$ (13)

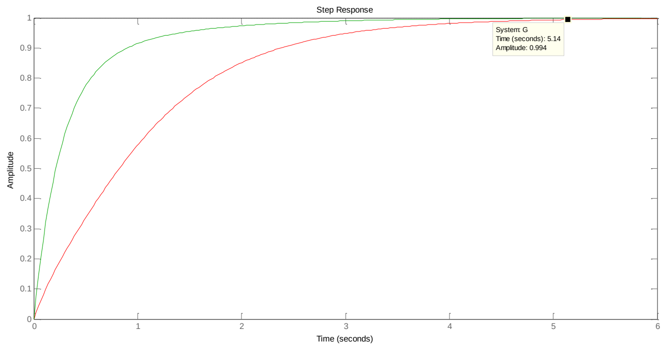


Figure 1: Step Response Comparison between Original System and Reduced System

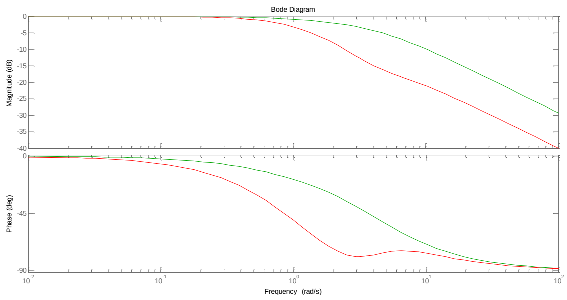


Figure 2: Bode Plots of Original System and Reduced System

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