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MODEL ORDER REDUCTION OF DISCRETE-TIME SYSTEMS USING BASIC CHARACTERISTICS AND MODIFIED CAUER FORM

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ABSTRACT

A computationally simple approach of model order reduction for single input single output (SISO) and linear time invariant discrete systems modeled in frequency domain is proposed in this paper. The denominator polynomial of the reduced order model is obtained by using the basic characteristics of the higher order system which are maintained in the reduced model while the coefficients of the numerator are obtained by using modified cauer form. This method is fundamentally simple and generates stable reduced models if the original high-order system is stable. The proposed method is illustrated with the help of the numerical example taken from the literature.

Keywords: *Model Order Reduction, Discrete-Time System, Higher Order System, Modified Cauer Form.*

Introduction

Higher order models are difficult to handle due to computational complexities and implementation difficulties and they are too complicated to be used in real time problems. It is therefore, desirable that a higher model is replaced with lower model. A large variety of methods of model order reduction are available in literature [1]-[5]. In spite of several methods available, no approach gives best result for all systems.

A very powerful method that involves simple algebraic calculations comprises continued fraction, moments matching and Padé approximation [2], [3]. However, the Padé method as a drawback that it may produce an unstable approximant for a given stable original system. Many methods such as Routh-approximation and Routh-Padéapproximants[4],[5]for continuous-time and [11], [12], [15] for discrete-time system have been suggested to obtain stable model. Model order reduction by matching Markov parameters, time moments and impulse energy approximation [7] have also suggested to ensure the stability of the reduced order model. Sumit Mondal [8] utilizing the basic characteristics of original system and pade approximation to get reduced order system. R. Parthasarathy and S. John [9] gives a very useful algorithm for reduction the order of high order system based on modified cauer form. It avoids finding the time moments and solving the Pade equation, whilst the reduced models still retains the initial time moments of full systems.

The modified cauer form can be used to determine the reduced numerator. In the proposed method, the denominator polynomial of reduced model is obtained using basic characteristics such as un-damped natural frequency of oscillations (ω_n), damping ratio(ε),settling time (T_s), peak overshoot (M_p) and peak time(t_p) while the coefficient of numerator is obtained using modified cauer form.

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Statement of the Problem

Let the transfer function of high order high order discrete –time stable system $G_o(z)$ of order 'n ' be

$$G_o(z) = \frac{N(z)}{D(z)} = \frac{a_0+a_1z+\cdots\ldots\ldots\ldots+a_{n-1}z^{n-1}}{b_0+b_1z+\cdots\ldots+b_nz^n} \quad (1)$$

wherea_i; $0\leq i \leq n-1$ and b_i; $0\leq i \leq n$known scalarconstants.

Let the transfer function of the reduced model of the order ' k'be

$$R(z)=\frac{N_k(z)}{D_r(z)}=\frac{c_0+c_1z+\cdots\ldots\ldots\ldots+c_{k-1}z^{k-1}}{d_0+d_1z+\cdots\ldots\ldots+d_rz^k} \quad (2)$$

where ;c_j; $0\leq j \leq k-1$ and d_j; $0\leq j \leq k$ are unknown scalar constants.

The aim of this paper is to realize the k^{th} order reduced model in the form of (2) from the original system (1) such that it retains the important features of the original high -order system.

Reduction Method

The reduction procedure for getting the k^{th}-order reduced models consist of the following two steps:

- **Step 1**

Apply bilinear transformation $z = \frac{1+w}{1-w}$, separately in the numerator and denominator polynomial of (1) using synthetic division. This convert $G_o(z)$ into G(w) as

$$N(w) = N(z)\Big|_{z=\frac{1+w}{1-w}} = \frac{N(w)}{(1-w)^{n-1}} = 0 \quad (3)$$

$$D(w) = D(z)\Big|_{z=\frac{1+w}{1-w}} = \frac{N(w)}{(1-w)^{n}} = 0 \quad (4)$$

- **Step 2**

Determination of the denominator polynomial for the k^{th} order reduced model using basic characteristics of original system by the following procedure

Firstly determine the basic characteristics of original system

Then assume damping ratio (ε)=0.99 for an aperiodic or almost periodic system, and number oscillations before the system settles=1

Determine the natural frequency(ω_n) using

$$w_n = \frac{4}{w * T_w}$$

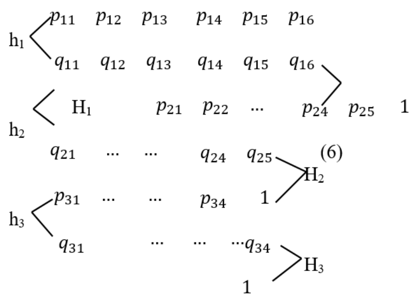
Obtain the reduced order denominator as:

$D_2(w) = w^2+2^* \varepsilon^* \omega_n+ \omega_n^2$ (5)

- **Step 3**

By applying the algorithm [9], the first 'k' quotients of modified Cauer form of continued fraction, viz. h_1, H_1, h_2, H_2 are evaluated.

Now a modified Routh array for k = 6 is built as given below:



where, the first two rows are formed from the denominator and numerator coefficients of $G_k(s)$ and the remaining entries in the array are obtained by the algorithm given in [9]. The sequence of computation is indicated by the arrows.

- **Step 5**

  Construct reduced order model

  $R(w) = \frac{N_k(w)}{D_k(w)}$ (7)

- **Step 6**

  Applying the inverse bilinear transformation

  $w = \frac{z-1}{z+1}$on$R(w)$ to obtain R(z) (8)

- **Step 7**

  Remove steady state error by evaluating the gain correction factorK=$\frac{G_o(z)}{R(z)}\Big|_{z=1}$

Method for Comparison

In order to check the quality of the proposed method the step response of original and reduced order is compared.

Numerical Example

The proposed method explains by considering numerical example, taken from the literature.

Example

Consider the original 4th order discrete system [16] is

$$G_o(z) = \frac{N(z)}{D(z)} = \frac{0.54377z^3\ - 0.40473 + 0.31921z - 0.216608}{z^4 - 1.361178z^3 + 0.875599z^2 - 0.55120z + 0.282145}$$

This system is to be reduced to 2nd order ROM. Applying bilinear transformation separately on Numerator and Denominator, using synthetic division, the equivalent continuous system becomes

$$G(w) = \frac{1.4879w^3 + 1.0778w^2 + 1.568w + 0.24526}{4.0707w^4 + 4.429w^3 + 5.9417w^2 + 1.2503w + 0.24476}$$

- **Step 1**

  Determination of numerator

  Settling time

  $t_w = 33.76667$

  ξ=0.99

  $$t_w = \frac{4}{\xi * w_n}$$

  $$w_n = \frac{4}{\xi * t_w} = \frac{4}{0.99 * 33.7667} = 0.1196$$

  $D_2(w) = w^2 + 2 * \xi * w_n w + w_n^2$

  $=w^2+2*0.99*0.1196w+(0.1196)^2$

  $= w^2+0.236w+0.0143$

- **Step 2**

  Modified Cauer form

0.24476	1.2503	5.9417	4.492	4.0707
0.24526	1.568	1.0778	1.4879	
−0.3145	4.866	3.003	1	
0.7132	−5.672	−3.390		

  h_1=0998, ,H_1=1.4879 h_2=-0.441, H_2=-2.390

  Inversion Table

0.0143	0.236	1
0.01427	1.4879	
	1	

  $N_2(s) = 1.4879s + 0.01427$

$$R_2(w) = \frac{1.1879w + 0.01427}{w^2 + 0.236w + 0.0143}$$

$R_2(z)=R_2(w)\,|\, w=\frac{z-1}{z+1}$

$= \frac{1.50217z-1.4736}{1.2503z^2-1.9714z+0.7783}$

After steady state correction

$K=\frac{G_4(z)}{R_2(z)}\,|_{z=1}=2.004$

$R_2(z)=\frac{3.01005z-2.9531}{1.2503z^2-1.9714z+0.7783}$

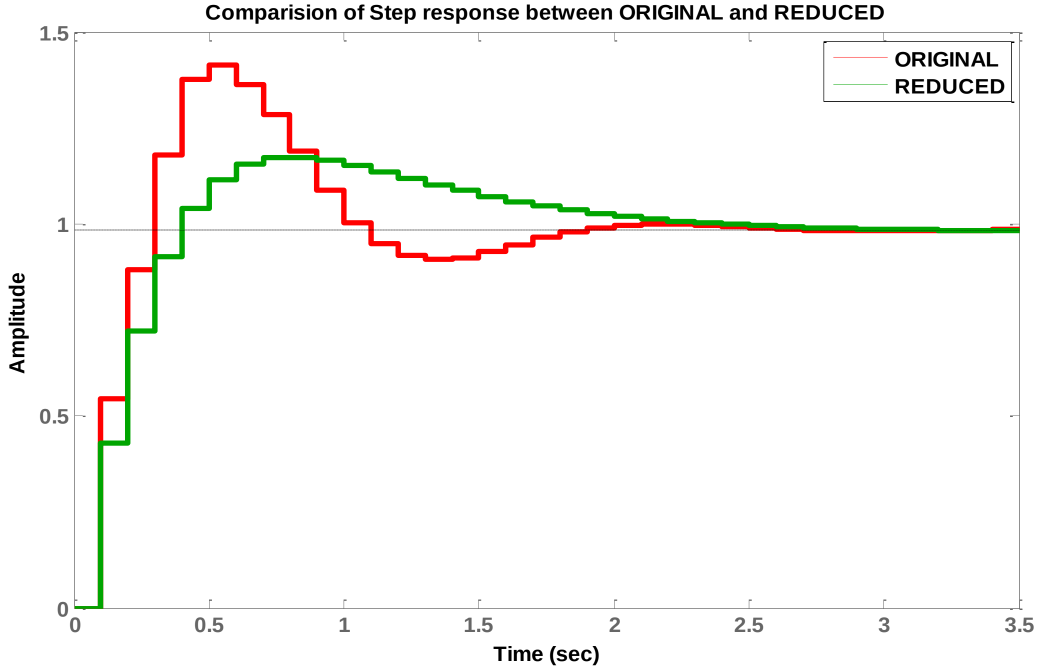


Fig. 1: Comparison of Step Responses of Original and Reduced Discrete System

Conclusion

In this paper, the original high-order discrete-time system is transformed to continuous-time system and its stable ROM is obtained using basic characteristics- modified cauer form. The denominator is obtained by retaining basic characteristics obtained the denominator, the numerators parameters are calculated by modified cauer form. Once the reduced-order model is obtained, its discrete-time model is derived. The advantages of proposed method are stable, simplicity, efficient and computer oriented. The proposed method has been explained with an example taken from the literature. The step responses of the original and reduced system of second order are shown in the Figure, from which we can conclude that proposed method is comparable in quality.

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