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TIME SERIES FORECASTING USING HOLT-WINTERS EXPONENTIAL

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ABSTRACT

Most of the industrial time series possessed seasonal pattern, such as in time series for apparel or toys. Hence, seasonal forecasting occupied prominent importance. This research paper depicts triple exponential smoothing method for seasonal time series data.

***KEYWORDS**: Seasonal Forecasting, Apparel or Toys, Seasonal Time Series Data, Seasonal Pattern.*

Introduction

Systematic projections about the future performance derived from the historical and current data is known as forecasting. Forecasting is used where precision of a decision is unknown Forecasting helps in the reduction of risks by making available the information about the future outcome. For the time series to be forecasted, the first step is to capture the data and then to select a model for forecasting. Various statistical and graphical techniques may be useful in the selecting the apt process. Specification of a model would be the next step after selecting the model, which involves selection of the variables, selection of the form of the equation of relationship, and evaluating the values of the parameters on that equation. After the model's specification, its performance characteristics must be verified or validated by comparison of its forecasts with historical data. Error measures such as MAPE (Mean absolute percentage error), RAE (Relative absolute error) MSE (Mean Square Error) may be used for validating the model. Selecting suitable error measure has an important effect on the conclusions about which of a set of forecasting methods is most appropriate.

Time-series forecasting's goal is to separate the pattern from the error by understanding the pattern's trend. Various time series forecasting methods are there in practice such as the Moving Averages method, Linear Regression with Time, Exponential Smoothing etc. This research paper presents report on the triple Exponential Smoothing technique on time series that possess seasonality.

Basic Terminologies

- **Additive and Multiplicative Seasonality**

Seasonality is often found in time series data, which can be defined as the tendency of time-series data to possess behavior that iterates itself every L periods, where L is the season length in periods. Hence the season is used to denote the time period before the beginning of the behavior to repeat itself. For example, the peak can probably be seen in annual sales of toys in the months of November and December, and during the summer (with a much smaller peak). The relative amount of increase in sales during December may slowly change from year to year, as this pattern is going to repeat every year. For example, the sales for a specific toy in the month of December may increase by 1million dollars every year. Hence, we count on our estimate that for every December the amount of 1 million dollars (over the respective annual average) to adjust for the seasonality factor which in this case is additive.

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The sales for a specific toy may increase by 40% during the month of December that is the increment by 1.4, so when the sales for the toys are generally slow, then the absolute (dollar) increase in sales will be relatively slow (but the percentage will be constant).

- **Linear, Exponential, and Damped Trend**

To continue with the toy example above, the sales for a toy can show a linear upward trend (e.g., each year, sales increase by 1 million dollars), exponential growth (e.g., each year, sales increase by a factor of 1.3), or a damped trend (during the first year sales increase by 1 million dollars; during the second year the increase is only 80% over the previous year, i.e., $800,000; during the next year it is again 80% over the previous year, i.e., $800,000 * .8 = $640,000; etc.).

- **Seasonality Index (SI)**

Seasonality Index of a period denotes how much this period deviates from the annual average.

Stationarity

To solve forecasting problems, a stationary condition needs to be met.

- **First Order Stationary**

If the expected value of X(t) remains same for all t it is known as first order stationary .

- **Second Order Stationary**

If the first order stationary and covariance between X(t) and X(s) is function of length (t-s) only then it is known as second order stationary

Exponential Smoothing

The process for continually iterating a forecast taking into consideration the more recent data then it is known as exponential smoothing. Exponential Smoothing placed exponentially increasing weights to the recent observations than the older ones.

- **Single Exponential Smoothing**

This is also called as simple exponential smoothing. Simple smoothing is used for shorter-range forecast, usually just one month into the future, assuming that the data fluctuates around a mean that is quite stable.

The formula for simple exponential smoothing is:

$$S_t = \alpha * X_t + (1 - \alpha) \_* S_{t-1} \quad (1)$$

Each new smoothed value (forecast) is computed as the weighted average of the current observation and the previous smoothed observation when applied recursively to each successive observation in the series. Thus, in effect, each smoothed value is the weighted average of the previous observations, where the weights decrease exponentially depending on the value of parameter (α). If it is equal to 1 (one) then the previous observations are ignored entirely; if it is equal to 0 (zero), then the current observation is ignored entirely, and the smoothed value consists entirely of the previous smoothed value (which in turn is computed from the smoothed observation before it, and so on; thus all smoothed values will be equal to the initial smoothed value S0). In-between values will produce intermediate results.

Initial Value

The initial value of S_t has a prominent role in computing all the subsequent values. Setting it to y_1 is one method of initialization.

- **Double Exponential Smoothing**

This method is useful when the time-series data shows a trend. In this two components needs to be updated for each period: - level and trend, where level is a smoothed estimate of the value of the data at the end of each period whereas trend depicts a smoothed estimate of average growth at the end of each period. The formula is as follows:

$$S_t = \alpha * y_t + (1 - \alpha) * (S_{t-1} + b_{t-1}) \quad 0 < \alpha < 1 \quad (2)$$

$$b_t = \gamma * (S_t - S_{t-1}) + (1 - \gamma) * b_{t-1} \quad 0 < \gamma < 1 \quad (3)$$

Initial Values

Various methods are available to select an initial values for S_t and b_t.

S_1 is set to y_1

Three suggestions for b_1

$b_1=y_2-y_1$

$b_1 = [(y_2 – y_1) + (y_3 – y_2) + (y_4 – y_3)]/3$

$b_1 = (y_n – y_1)/(n − 1)$.

- **Triple Exponential Smoothing**

This method is applicable when both the trend and seasonality exhibit by the time-series data. We need to add a third parameter to handle seasonality.

- **Multiplicative Seasonal Model**
- **Additive Seasonal Model**

The rest of the report focuses on these two models.

Multiplicative Seasonal Models

This model is applied when Multiplicative seasonality is possessed by the time-series data as discussed in Section 2.1.

- **Overview**

In this model, we assume that the time series is represented by the model

$y_t = (b_1 + b_2t) S_t +€_t$ (4)

Where

b_1 is the base signal also called the permanent component

b_2 is a linear trend component

S_t is a multiplicative seasonal factor

$€_t$ is the random error component

Let the length of the season be L periods.

The seasonal factors are defined so that they sum to the length of the season, i.e.

$$\sum_{1\le t\le L} S = L$$

The trend component b_2 if deemed unnecessary maybe deleted from the model.

- **Application of the Model**

It is applied to a time series displaying multiplicative seasonality and model is apt for a data in where the amplitude of the seasonal pattern is proportional to the average level of the series.

- **Details**

This part depicts about the equations used in the model along with the initial values .

- **Notation used**

Let the current deseasonalized level of the process at the end of period T be denoted by R_T.
At the end of a time period t, let
$\check{R}_t$ be the estimate of the deseasonalized level.
$\check{G}_t$ be the estimate of the trend
$\dot{S}_t$ be the estimate of seasonal component (seasonal index)

- **Procedure for updating the estimates of model parameters**
  - **Overall Smoothing**

$\check{R}_t= α(y_{t/} \dot{S}t−L) + (1 − α) *(\check{R}_{t-1} + \check{G}_{t-1})$ (6)

where 0 < α< 1 is a smoothing constant.

Dividing y_t by $\dot{S}_{t-L}$, which is the seasonal factor for period T computed one season (L periods) ago, deseasonalizes the data so that only the trend component and the prior value of the permanent component enter into the updating process for R_T .

  - **Smoothing of the Trend Factor**

$\check{G}_t = β * (\dot{S}_t− \dot{S}_{t-1}) + (1 − β) *\check{G}_{t-1}$ (7)

where 0 < β < 1 is a second smoothing constant.

The estimate of the trend component is simply the smoothed difference between two successive estimates of the deseasonalized level.

- **Smoothing of the Seasonal Index**

  $\dot{S}_t = \gamma (y_t / \dot{S}_t) + (1 - \gamma) * \dot{S}_{t-L}$ (8)

  where $0 < \gamma < 1$ is the third smoothing constant.

  The estimate of the seasonal component is a combination of the most recently observed seasonal factor given by the demand yt divided by the deseasonalized series level estimate Rt and the previous best seasonal factor estimate for this time period. Since seasonal factors represent deviations above and below the average, the average of any L consecutive seasonal factors should always be 1. Thus, after estimating St, it is good practice to re normalize the LP most recent seasonal factors such that $\sum_{i=t-q+1}^{t} Si = q$

- **Value of Forecast**
  - **Forecast for the Next Period**

    The forecast for the next period is given by:

    $yt = (\check{R}_{t-1} + \check{G}_{t-1})\, \dot{S}_{t-L}$ (9)

    Note that the best estimate of the seasonal factor for this time period in the season is used, which was last updated L periods ago.

  - **Multiple-step-ahead forecasts (for T < q)**

    The value of forecast T periods hence is given by:

    $Y_{t+T} = (\check{R}_{t-1} + T * \check{G}_{t-1})\, \dot{S}_{t+T-L}$ (10)

- **Initial Values of Model Parameters**

  As a rule of thumb, a minimum of two full seasons (or 2L periods) of historical data is needed to initialize a set of seasonal factors. Suppose data from m seasons are available and let $\bar{x}_j$, j = 1, 2, · · · ,mL denote the average of the observations during the j^{th} season.

  - **Estimation of Trend Component**

    Estimate the trend component by:

    $\check{G}_{0=(} y_m - {}_{y1)/}(m - 1)L$ (11)

  - **Estimation of Deseasonalized Level**

    Estimate the deseasonalized level by:

    $\check{R}_0 = x_1 - (L/2)\check{G}_0$ (12)

  - **Estimation of seasonal components**

    Seasonal factors are computed for each time period t = 1, 2, · · · ,mL as the ratio of actual observation to the average seasonally adjusted value for that season, further adjusted by the trend; that is,

    $\dot{S}_{t=}x_t / (x_i - [(L + 1)/2 - j]\check{G}_0)$ (13)

    where x_i is the average for the season corresponding to the t index, and j is the position of the the period t within the season. The above equation will produce m estimates of the seasonal factor for each period.

    $\dot{S}_{t=i}/m \sum_{k=0}^{m-1} \mathbf{St + kL}$ t = 1, 2, · · · ,L (14)

    $\dot{S}_t(0) = \dot{S}_t/(L / \sum_{t=1}^{L}$ t = 1, 2, · · · ,L (15)

Additive Seasonal Model

This model is useful when the time-series datashows Additive seasonality.

- **Overview**

  In this model, we assume that the time series is represented by the model

$y_t = b_1 + b_2 t + S_t + €t$ (16)

Where

b_1 is the base signal also called the permanent component

b_2 is a linear trend component

S_t is a additive seasonal factor
€t is the random error component
Let the length of the season be L periods.

The seasonal factors are defined so that they sum to the length of the season, i.e.

$$\sum_{1<t<L} St = 0 \tag{17}$$

The trend component b_2 if deemed unnecessary maybe deleted from the model.

- **Application of the Model**

It is applicable when a time series is displaying additive seasonality and the model is apt for a data where the amplitude of the seasonal pattern is independent of the average level of the data.

- **Details**

This part discusses about the forecasting equations used along with the initial values to be used for the parameters.

- **Notation Used**

Let the current deseasonalized level of the process at the end of period T be denoted by R_T .

At the end of a time period t, let

R_t be the estimate of the deseasonalized level.

G_t be the estimate of the trend

$\bar{S}_t$ be the estimate of seasonal component (seasonal index)

- **Procedure for Updating the Estimates of Model Parameters**
  - **Overall Smoothing**

    $R_t = \alpha(y_t - S_{t-L}) + (1 - \alpha) * (R_{t-1} + G_{t-1})$ (18)

    where $0 < \alpha < 1$ is a smoothing constant.

    Dividing y_t by S_{t-L}, which is the seasonal factor for period T computed one season (L periods) ago, deseasonalizes the data so that only the trend component and the prior value of the permanent component enter into the updating process for R_T.
  - **Smoothing of the Trend Factor**

    $G_t = \beta * (S_t - S_{t-1}) + (1 - \beta) * G_{t-1}$ (19)

    where $0 < \beta < 1$ is a second smoothing constant.

    The estimate of the trend factor is the smoothed difference between two successive estimates of the deseasonalized level.
  - **Seasonal Index Smoothing**

    $S_t = \gamma (y_t - S_t) + (1 - \gamma) * S_{t-L}$ (20)

    where $0 < \gamma < 1$ is the third smoothing constant.

    The estimate of the seasonal component is a combination of the most recently observed seasonal factor given by the demand y_t divided by the deseasonalized series level estimate R_t and the previous best seasonal factor estimate for this time period. Since seasonal factors represent deviations above and below the average, the average of any L consecutive seasonal factors should always be 1. Thus, after estimating St, re normalizing the L most recent seasonal factor is a good practice.

    L most recent seasonal factors such that $\sum_{i=t-q+1}^{t} Si = q$
- **Value of Forecast**

The next period's forecast is given by:

$y_t = R_{t-1} + G_{t-1} + S_{t-L}$ (21)

the best estimate of the seasonal factor was last updated L periods ago.

Experimental Results

Variations of the additive and multiplicative exponential smoothing techniques were applied to some standard time series data, the results of which are discussed in this section.

- **Test Cases**

The following tests were carried out:

Multiplicative Model

Two varied models are used:

- **Non-adaptive**

For establishing the parameters and building the model data of the first two years is used in the non-adaptive technique. Forecasts are made for the remaining data using these parameters, hence once initialized, the values of α, β and γ are not modified later. MAPE is used as a criterion for selection of the parameters.

  - As the parameters have been established the forecasting can proceed without any delay in re-computation of the parameters the parameters are initialized only once.
  - Past data need not be remembered is one of its advantage.

- **Adaptive**

In the adaptive technique, the original parameters are established using the initial first two year's data. However, the model keeps updating itself to the changes required in the process. After every two years the parameters are adjusted. All the new parameters are computed using

  - All available data till that point of time or
  - k most recent data.

All the past data needs to be stored for the first approach.

  - **Additive Mode**l

There are two variations of the model used. i.e. Adaptive and Non-Adaptive.

The value of L can vary between 6 and 24 for the unknown value of L

- **Summary of Test Results**

The test results are presented in this section. The tests were conducted on the following time series : display, processor, memory, battery and camera.(for mobile phones).

The results obtained from the multiplicative model are better results than those of the additive model for all the series.

The adaptive multiplicative model gives better results for the series like display as compared to the non-adaptive multiplicative model, while for series like processor, the non-adaptive multiplicative model gave better results.

Taking display and processor as examples, the results are summarized below.

  - **display** The results obtained by using Multiplicative Adaptive technique are summarized inTable1

Table 1: Adaptive Multiplicative Technique for Display

L	Look back	MAPE
12	24	142.84827
12	δ	129.24403
13	26	113.98809
13	δ	111.99094
15	30	114.87889
15	δ	119.090744
17	34	120.50779
17	δ	115.598305
18	36	120.56211
18	δ	117.90184
23	46	123.13986
23	δ	109.389824

The best result value of MAPE is obtained with an infinite look back for the value of L = 23,

For adaptive technique the MAPE is 109 while for the non-adaptive technique is 10431.5, which mean that the series is constantly updating. Consequently, the model parameters need to be constantly updated according to the newer data. Exponential smoothing technique doesn't appear to be the best forecasting technique for this data, as the value of MAPE is 109.

- **Display**

The multiplicative non-adaptive exponential smoothing technique gave better results as compared to the adaptive version of the same in this time-series. The value of MAPE obtained was 9.01809 with α= 0.2, β = 0.1 and γ= 0.1.

Multiplicative Adaptive technique on the time series data summary is shown in Table 2.

Table 2: Adaptive Multiplicative Technique for Processor

L	Look Back	MAPE
10	20	21.62986
10	δ	22.011831
12	24	18.493525
12	δ	18.515306
24	48	17.48018
24	δ	17.722464

The results indicate that the periodic re-evaluation of the parameters is not going to improve the results. The time series can possess fluctuations. It is possible that new set of parameters are affected by the fluctuations if the parameters are revaluated periodically.

Conclusion

The triple exponential smoothing is applied for the time-series where both trend and seasonality is possessed by a given data. The two widely applied models are Additive model for time series possessing additive seasonality and Multiplicative model for time series possessing Multiplicative seasonality. Model parameters, α, β and γ are initialized using the data of the first two years. The error measure used for selecting the best parameters is MAPE. In the adaptive model technique, these parameters are recursively updated from the most recent observed data. The main reason behind using the adaptive technique, instead of non-adaptive technique is that, the time series at some point can change its behavior and the model parameters must be able to adapt to this change in behavior.

Tests carried out on some standard time series data corroborated this assumption.

The value of L and the Look back size also play has prominent effect in the performance of the adaptive model; hence these parameters were also varied for forecasting the different time series. It was observed that the adaptive technique with a larger look back in general improved the results (as compared to the non-adaptive version). However, adaptive technique with a smaller look back often performed worse than the non-adaptive technique. This suggests that re-computing the model parameters based on only few of the most recent observations may be an unnecessary overhead which may lead to poor performance. Hence, the adaptive technique should be combined with a sufficiently large look back size in order to obtain good results.

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