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STUDY OF HEAT TRANSFER IN THE PRESENCE OF MHD FLOW OF RIVLIN-ERICKSEN FLUID PAST A MOVEABLE VERTICAL PLATE ENRICHED IN POROUS MEDIUM

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ABSTRACT

In this paper we study the effect of heat transfer with free convective flow of Rivlin-Ericksen fluid along a vertical semi-infinite moving permeable plate enriched in the porous medium with variable suction has been measured. Continuity and heat equations are solved and their particular results are depicted through graphs. In other hand we also calculated the Nusselt number and skin-friction coefficient are also measured and depicted through tables. It was concluded from the actual study that the permeability of the porous region supports the fluid velocity as well as fluid temperature which is caused by less obstacles in fluid flow.

Keywords: *MHD, Porosity, Rivlin-Ericksen Fluid.*

Introduction

The study of energy and heat transfer in the fluid flow through various geometries enriched in porous medium has numerous industrial and geophysical usages like dehydrating of porous solids, thermal insulations, cooling of electronics system, nuclear reactors, abstraction of crude oil etc.

Chamkha[1] deliberated the free convection flow through a vertical plate immersed in a porous medium with magnetic effects. In the similar outer conditions Eldabe[2] analyzed the flow of the non-Newtonian fluid in the presence of heat source.

Related problems of heat transfer has many applications in many processes and have consequently a substantial extent of attraction in recent years. Probable uses of this type of flow can be found in many manufacturing and industries. MHD boundary layer flow is of extensive attention due to its wide procedure in industrial technology and geothermal application, high temperature plasmas appropriate to nuclear fusion energy conversion, MHD power generation systems and liquid metals fluids.

Sahinet al.[3] discussed heat and mass transfer by mixed convection MHD flow along a porous plate with chemical reaction in the presence of heat source. Then Sharma et al.[4] extended same study the for visco-elastic fluid (Walter's liquid model-B).

In view of that Daleepet al.[5] discussed the bounds for complex growth rate in the flow of Rivlin-ericksen viscoelastic fluid in porous medium. Rana[6] extended their work to illustrate the thermal instability of same rotating fluid saturated with suspended dust particles. In recent years, Popoola et al[7] analyzed the heat and mass transfer on MHD viscoelastic fluid flow in the presence of thermal diffusion and chemical reaction. Then Ravikumar et al.[8] extend the work of Kumar et al.[9] by considering the same flow with MHD and variable suction.

This type of work has not been discussed in Rivlin-Ericksen fluid flow. In the present paper the effect of heat transfer with magnetic effects are discussed and result depicted through graphs.

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Mathematical Formulation

Consider a two dimensional unsteady flow of viscous immiscible incompressible electrically conducting Rivlin-Ericksen fluid flow along a moveable vertical semi-infinite porous plate in entrenched in a uniform porous medium subjected to an constant transverse magnetic field.

Let $\bar{x}$ -axis be vertically upward along the plate and in the direction of flow, and $\bar{y}$ -axis taken normal to it.

These are, the governing equations of the fluid flow are given as

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \upsilon\frac{\bar{u}}{\bar{K}p} - \frac{\sigma B_0^2}{\rho}\bar{u} - \beta\left(\frac{\partial^3 \bar{u}}{\partial \bar{t}\,\partial \bar{y}^2} + \bar{v}\frac{\partial^3 \bar{u}}{\partial \bar{y}^3}\right), \tag{2}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\alpha}{\rho Cp}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\sigma B_0^2}{\rho Cp}\bar{u}^2 + \frac{\mu}{\bar{K}p\rho Cp}\bar{u}^2, \tag{3}$$

Then the boundary conditions for the velocity and temperature fields are given as follows:

$$\bar{y} = 0: \quad \bar{u} = \bar{u}_p, \qquad \bar{T} = \bar{T}_W + \in\left(\bar{T}_W - \bar{T}_\infty\right)e^{\bar{n}\,\bar{t}},$$

$$\bar{y} \to \infty : \bar{u} \to \bar{U}_\infty = U_0\left(1 + \in e^{\bar{n}\,\bar{t}}\right), \qquad \bar{T} \to T_\infty, \tag{4}$$

By equation (1) the suction velocity at the surface of the plate is varying with time only. We assume

$$\bar{v} = -V_0\left(1 + \in A e^{\bar{n}\,\bar{t}}\right), \tag{5}$$

where V_0 is the mean suction velocity, A is a real constant and $\in A$ is lesser than unity $\left[\in A << 1\right]$. For the outside region of the boundary layer, equation (2) follows

$$-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} = \frac{d\bar{U}_\infty}{d\bar{t}} + \frac{v}{\bar{k}}\bar{U}_\infty + \frac{\sigma}{\rho}B_0^2\,\bar{U}_\infty. \tag{6}$$

Method of Solution

Introducing the following non-dimensional quantities,

$$u = \frac{\bar{u}}{U_0}, \quad v = \frac{\bar{v}}{V_0}, \quad y = \frac{V_0\,\bar{y}}{v}, \quad U_\infty = \frac{\bar{U}_\infty}{U_0}, \quad U_p = \frac{\bar{u}_p}{U_0}, \quad t = \frac{\bar{t}\,V_0^2}{v}, \quad n = \frac{\bar{n}\,v}{V_0^2},$$

$$\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_W - \bar{T}_\infty}, \quad k = \frac{\bar{K}V_0^2}{v^2}, \quad \Pr = \frac{v\,\rho Cp}{k}, \quad M = \frac{\sigma B_0^2\,v}{\rho V_0^2},$$

$$Rc = \frac{\beta V_0^2}{v^2}, \quad \gamma = \frac{K_l\,\upsilon}{V_0^2}, \quad Ec = \frac{\upsilon^2}{Cp\left(\bar{T}_W - \bar{T}_\infty\right)}, \quad N = M + \frac{1}{K}, \tag{7}$$

into the equations (1)-(3) with equations (5) & (6), we get

$$\frac{\partial u}{\partial t}-\left(1+\in Ae^{nt}\right)\frac{\partial u}{\partial y}=\frac{dU_\infty}{dt}+\frac{\partial^2 u}{\partial y^2}+N\left(U_\infty-u\right)-Rc\left[\frac{\partial^3 u}{\partial t\,\partial y^2}-\left(1+\in Ae^{nt}\right)\frac{\partial^3 u}{\partial y^3}\right], \tag{8}$$

$$\frac{\partial \theta}{\partial t}-\left(1+\in Ae^{nt}\right)\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+\frac{M}{\Pr}u^2+\frac{Ec}{\kappa}u^2, \tag{9}$$

the boundary conditions (4) are reduced into

$$\begin{aligned} &y=0: \quad u=U_p, \quad \theta=1+\in e^{nt} \\ &y\to\infty: \quad u\to U_\infty, \quad \theta\to 0. \end{aligned} \tag{10}$$

Since the equations (8) &(9) are the set of coupled non-linear partial differential equations which cannot be solved analytically. In view of that the following expression is used to separate the steady and unsteady parts of velocity and temperature distribution:

$$u(y,t)=u_0(y)+\in u_1(y)e^{nt}+O\left(\in^2\right)+\cdots, \tag{11}$$

$$\theta(y,t)=\theta_0(y)+\in \theta_1(y)e^{nt}+O\left(\in^2\right)+\cdots, \tag{12}$$

where u_0,θ_0 denote steady parts and u_1,θ_1 denote unsteady parts. Equations (11) & (12) are substituted into the equations (8) & (9) and then harmonic and non-harmonic terms are equated. Here we are assuming to neglect the higher order term of $\in$.

Zeroth-order equations

$$Rc\,u_0^{***}+u_0^{**}+u_0^{*}-N u_0=-N, \tag{13}$$

$$\theta_0^{**}+\theta_0^{*}\Pr=-M u_0^2-\frac{Ec}{\kappa}u_0^2, \tag{14}$$

First-order Equations

$$Rc\,u_1^{***}+\left(1-Rc\,n\right)u_1^{**}+u_1^{*}-N u_1-n u_1=n-N-Rc\,u_0^{***}-A u_0^{*}, \tag{15}$$

$$\theta_1^{**}+\Pr\theta_1^{*}-n\Pr\theta_1=-A\Pr\theta_0^{*}-2Mu_0\,u_1-2\frac{Ec\Pr}{\kappa}u_0\,u_1, \tag{16}$$

where star (*) denotes the differentiation with respect to y.

The corresponding boundary conditions are reduced into

$$\begin{aligned} &y=0: \quad u_0=U_p, \; u_1=0, \quad \theta_0=1, \quad \theta_1=1, \\ &y\to\infty: \quad u_0=1, \quad u_1=1, \quad \theta_0\to 0, \; \theta_1\to 0. \end{aligned} \tag{17}$$

Equations (13) and (15) are third order coupled differential equations, however only two boundary conditions are available. Assuming $Rc\neq 0$ then u_0 and u_1 can be expanded in the powers of Rc as given below

$$u_0=u_{01}+Rc\,u_{02}+O\left(Rc^2\right), \tag{18}$$

$$u_1=u_{11}+Rc\,u_{12}+O\left(Rc^2\right). \tag{19}$$

Using equations (18) & (19) into the equations (13) and (15); equating the coefficients of like powers of Rc and neglecting the higher order of $Rc,$ we get

Zeroth-order Equations

$$u_{01}^{**} + u_{01}^{*} - N u_{01} = -N, \quad (20)$$

$$u_{02}^{**} + u_{02}^{*} - N u_{02} = -u_{01}^{***}, \quad (21)$$

First-order Equations

$$u_{11}^{**} + u_{11}^{*} - (N+n) u_{11} = -A u_{01}^{*} - n - N, \quad (22)$$

$$u_{12}^{**} + u_{12}^{*} - (N+n) u_{12} = -u_{11}^{***} + n u_{11}^{**} - u_{01}^{***} - A u_{02}^{*}, \quad (23)$$

Now the corresponding boundary conditions are reduced to

$$y = 0: \quad u_{01} = U_p, \quad u_{02} = 0, \quad u_{11} = 0, \quad u_{12} = 0,$$
$$y \to \infty: \quad u_{01} = 1, \quad u_{02} = 0, \quad u_{11} = 1, \quad u_{12} = 0. \quad (24)$$

The equations (20) to (23) are ordinary second order linear differential equations. Through straight forward calculation, we get the result.

Skin-friction Coefficient

The coefficient of skin-friction at the plate is given by

$$C_f = \left[\frac{\tau_w}{\rho v_0^2} \right]_{y=0} = \left(\frac{\partial u}{\partial y} \right)_{y=0} = \left(\frac{\partial u_0}{\partial y} \right)_{y=0} + \in e^{nt} \left(\frac{\partial u_1}{\partial y} \right)_{y=0}. \quad (25)$$

Table 1: The numerical values of coefficient of skin-friction at the plate for different values of physical parameters when n=1 and t=1.

S.No.	Rc	M	Kp	C_f
1.	1	3	1.5	2.87
2.	3	3	1.5	0.2581
3.	1	3	1.5	1.554
4.	1	5	1.5	0.6731
5.	1	3	2.5	1.7895
6.	1	3	1.5	4.0408
7.	1	3	1.5	0.7839

Nusselt Number

The rate of heat transfer in term of Nusselt number at the plate is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y} \right)_{y=0} = -\left(\frac{\partial \theta_0}{\partial y} + \in e^{nt} \frac{\partial \theta_1}{\partial y} \right)_{y=0}. \quad (26)$$

Table 2: The numerical values of coefficient of Nusselt number at the plate for different values of physical parameters when n=1 and t=1.

S.No.	Rc	M	Kp	Ec	Pr	Nu
1.	1	3	1.5	0.5	0.71	-63.98
2.	3	3	1.5	0.5	0.71	-201.67
3.	1	3	1.5	0.5	0.71	-60.21
4.	1	5	1.5	0.5	0.71	-117.41
5.	1	3	2.5	0.5	0.71	-57.00
6.	1	3	1.5	0.5	0.71	-122.33
7.	1	3	1.5	0.5	0.71	-71.29
8.	1	3	1.5	1	0.71	-64.52
10.	1	3	1.5	0.5	7.0	103.64

Result & Discussion

We get the numerically result for velocity and temperature for various values of porosity parameter, Hartman number, mass buoyancy, Schmidt number, Eckert number and Prandtl number which describe the flow characteristics and the results are reported in terms of graphs.

Fig.(2) shows the effects of magnetics field parameter and permeability on the velocity profile. It is seen that velocity increase with the upsurge of Hartman number through the boundary layer. Reverse behaviour is observed in the case of permeability parameter.

Fig.(3) exhibits that the effects of visco-elastic parameter of the Rivlin-Ericksen fluids, It is also seen that their presence reducing the velocity field at all points. The same result of this agreement are found by Ravikumar et al.[9].

We observed from fig.(4) Due to the increase of intensity of magnetic field; fluid temperature is increasing rapidly throughout the flow region and opposite behaviour we seen for the porosity parameter.

Consequence of visco-elastic parameter, are shown from fig.(5). Due to upsurging in the elastic parameter of the fluid, the fluid temperature is decling continuously in the flow region.

From fig.(6) it is seen that upsurge in Prandtl number increasing the fluid temperature and Eckert number, which is the characteristic of heat dissipation enhancing the fluid temperature throughout the fluid region.

The impact of all physical parameters on skin-friction coefficient and Nusselt number for different values of the pertinent parameters is displayed in Tables 1 & 2. It can be noted from that the table-1, all the values decline.

On careful study of table-2, it is observed that the Nusselt number increases with an increase of permeability parameter while decreases for other parameters.

Conclusions

In this research article, the effects of MHD free convective flow of Rivlin-Ericksen fluid past a vertical porous plate has been considered. The resultant coupled non-linear differential equations are solved with suitable boundary conditions. The effects of various physical parameters which are regulating the fluid flow are discussed under results and discussion. By using the obtained results the following conclusions are made: The flow of Rivlin-Ericksen fluid is highly effected by elasticity parameter. The high elastic fluid flow is slow under the other stable environment.

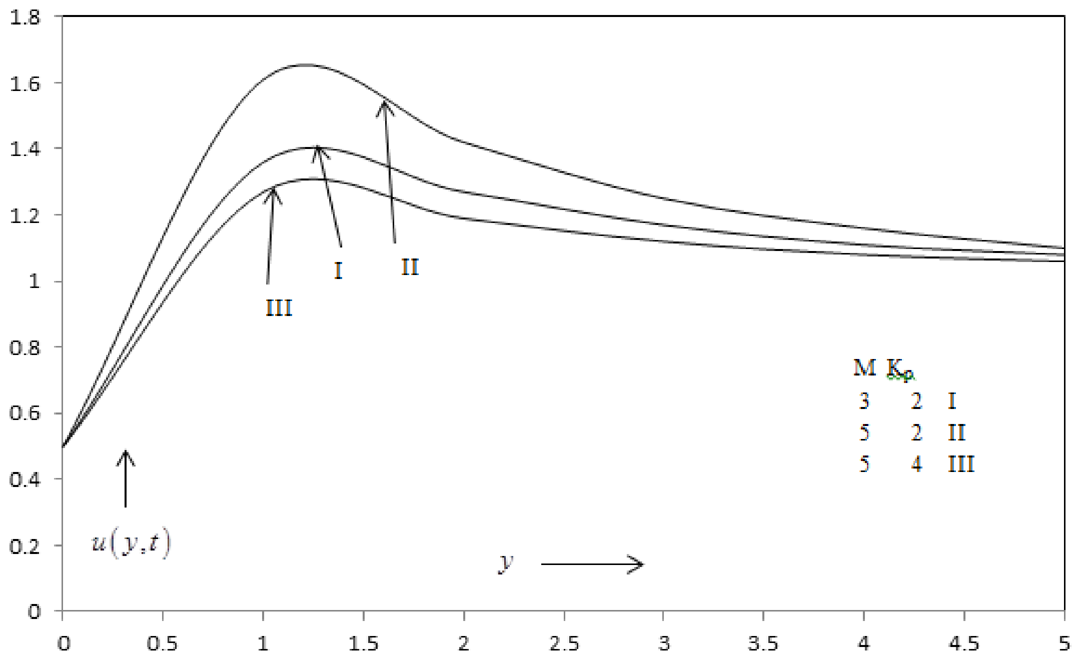

Fig.2 Velocity distribution versus y when Rc=1.5, A=0.5, κ=0.5, ε=0.25.

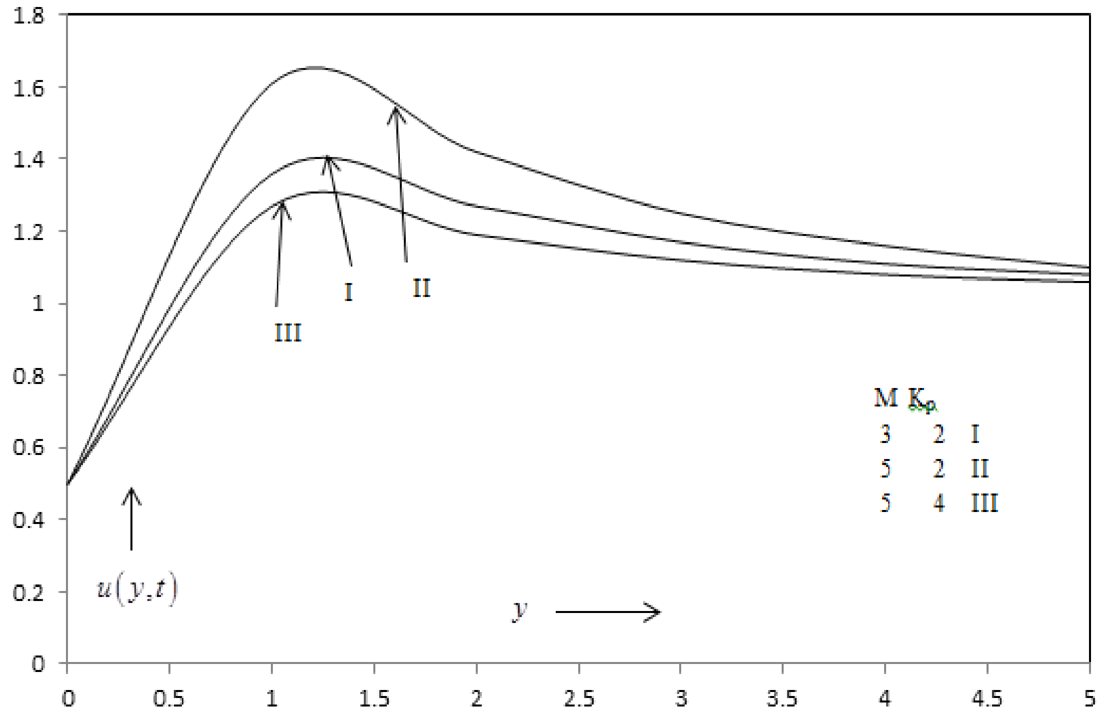

Fig. 3: Velocity distribution versus y when A=0.5, κ=0.5, Kp=1.5, M=2,and ε=0.25.

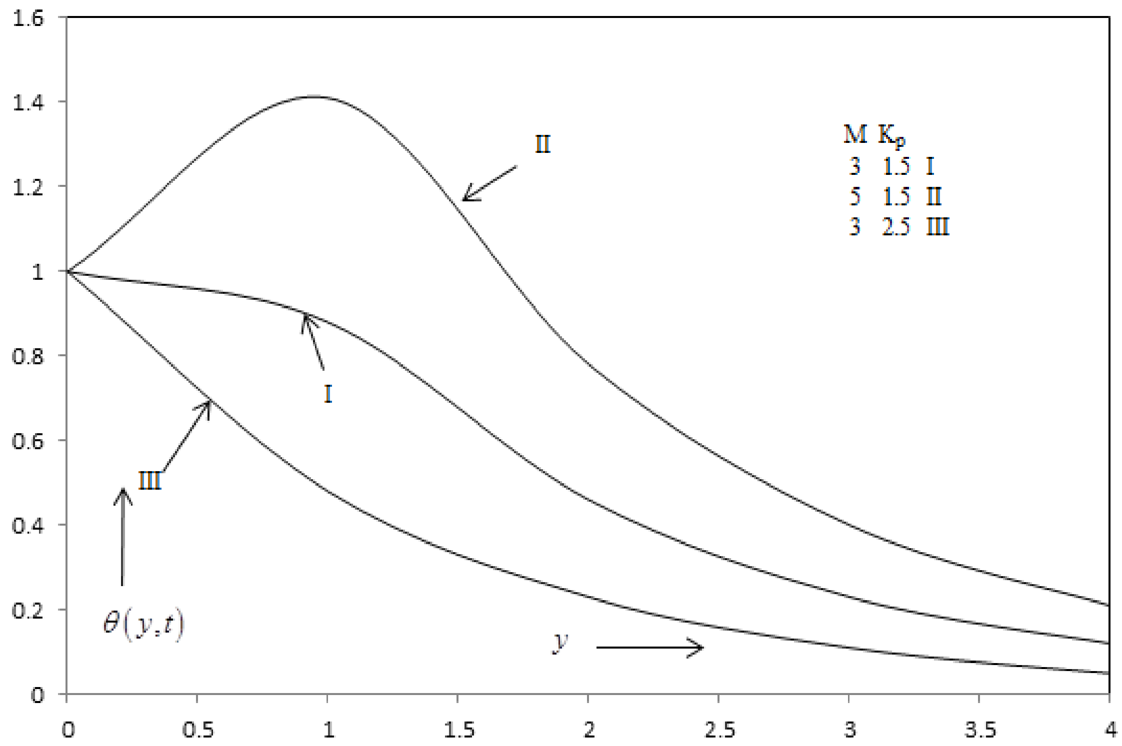

Fig. 4: Temperature distribution versus y when Rc=1.5, A=0.5, κ=0.5, Ec=1, ε=0.25 and Pr=0.71

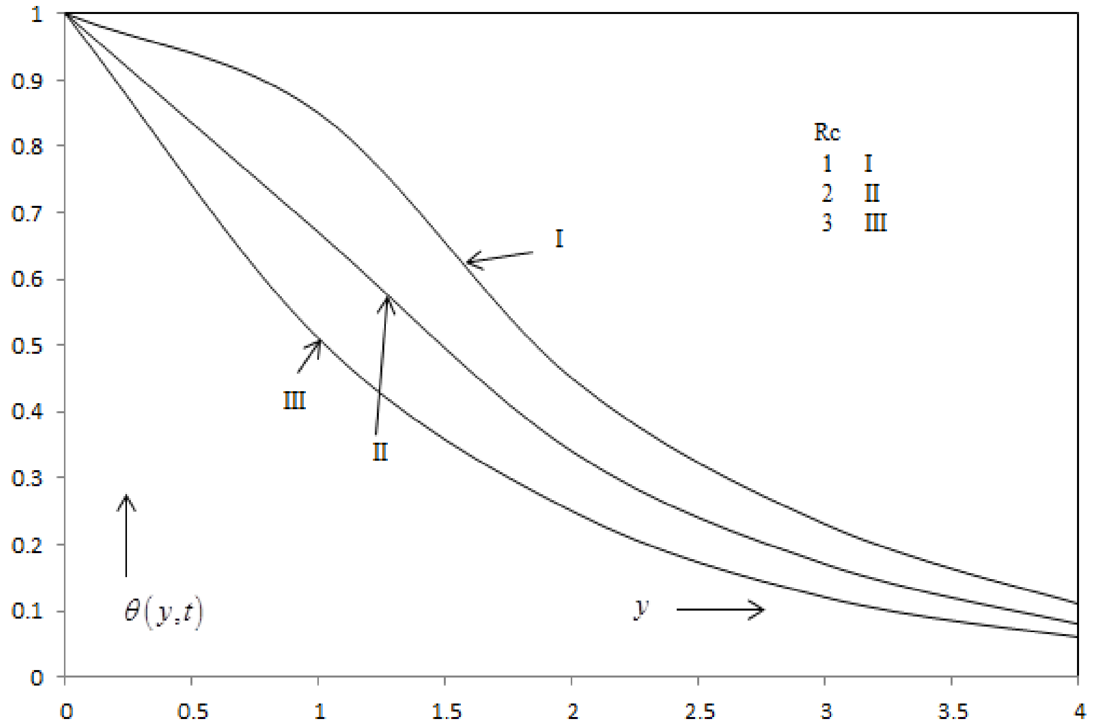

Fig. 5: Temperature distribution versus y when M=2, A=0.5, κ=0.5, Kp=1.5 Ec=1,ε=0.25 and Pr=0.71

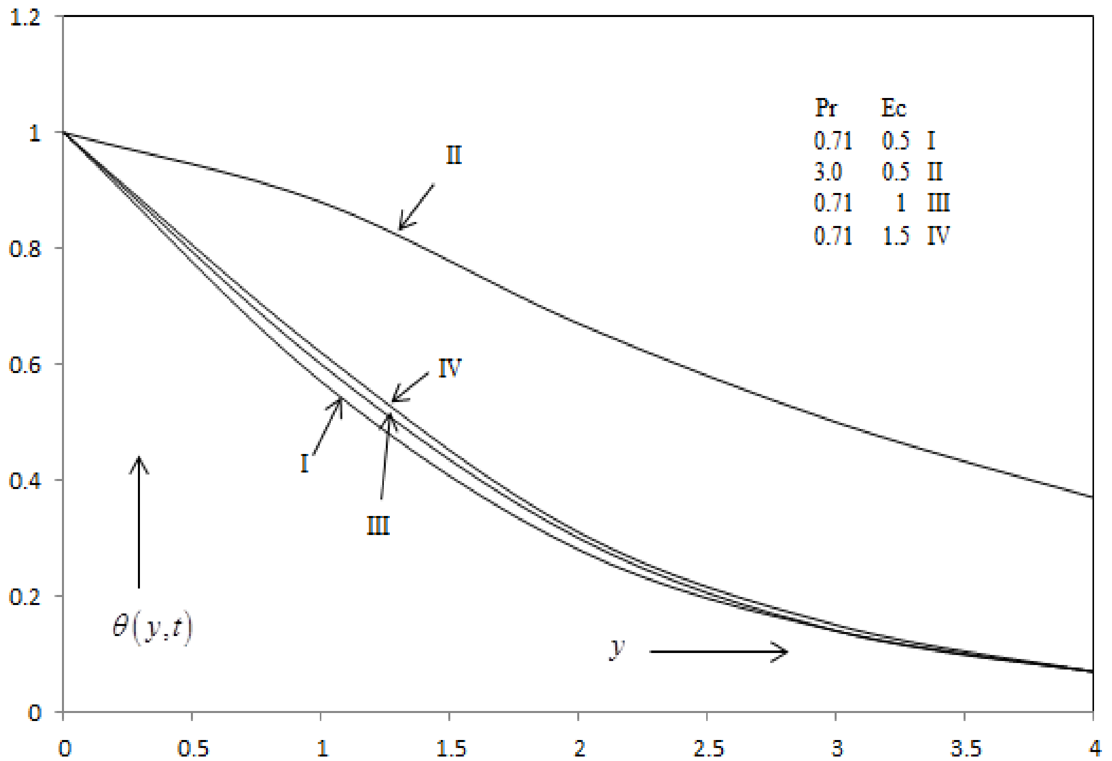

Fig.6: Temperature distribution versus y when Rc=1.5, A=0.5, κ=0.5, M=3, Kp=1.5, and ε=0.25

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