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MATHEMATICAL AND GRAPHICAL STUDY OF MEAN QUEUE LENGTH OF A QUEUEING SYSTEM HAVING COMBINATION OF SIX SERVERS CENTRALLY LINKED WITH A COMMON SERVER

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ABSTRACT

The present paper deals with the mathematical and graphical study of mean queue length of a queueing system having combination of six servers out of which one server is centrally linked with other five server. Mean queue lengths of each six servers and the whole system has been derived by solving steady state equations using generating function technique.

Keywords: *Centrally Linked Server, Mathematical and Graphical, Mean Queue Length, Steady State Equations, Generating Function Technique.*

Introduction

Feedback queues are those queues which are formed due to rejoining of the queuing system by the unsatisfied customers who have already availed the service. In real life feedback queues are observed generally in manufacturing concerns, offices, in hospitals etc. Many researchers have done a lot of work on queuing models considering various aspects such as bulk service, impatient customers, cyclic queues, batch arrivals, reneging, blocking. Study on queuing models considering the concept of feedback has also been done by many researchers. Jackson (1957) did a lot of work in the field of cyclic queues with feedback. Multi-server and multiclass queuing models have been discussed by a good number of researchers such as Jianghua and Jinting (2006), Houdt et al. (2008), Goswami and Pandit (2011), Sundri and Srinivasan (2012), Aristotles and Endah (2013), Ibe and Isijola (2014), Morozov (2014), Kang (2015), Yanfeng and Christos (2015), Zadeh (2015), Atar and Mendelson (2016), Avrachenkov et al. (2016), Jansen et al. (2016), Reed and Zhang (2017), Antonioli et al. (2018), Ginting et al. (2018).

Kusum et al. (2010) worked on queues with feedback wherein three servers have been considered wherein a customer may go back to his/her preceding service channel or moving forward to immediate next service channel, except from the third server. Gupta Deepak, Naveen Gulati (2011) studied the steady state behaviour of a network queue model with biserial and parallel channels linked wiyh a common server. Meena Gupta, Deepak Gupta (2012) studied the concept of steady state solutions of multiple parallel channels in series and non-serial multiple parallel channels both in balking and reneging. Gupta Deepak, Naveen Gulati (2011)studied the steady state behaviour of a network queue model with biserial and parallel channels linked wiyh a common server. Meena Gupta, Deepak Gupta (2012) studied the concept of steady state solutions of multiple parallel channels in series and non-serial multiple parallel channels both in balking and reneging. Surender Kumar and Gulshan Taneja

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(2018) analyzed queueing model with chances of revisit of customer at most twice to any of the three servers. Harminder Singh et al (2019) worked on the analysis of network queue model comprised of parellel channels centrally linked with a common server by using the laplace transformation.

Santosh Kumari (2021) found the Mean queue lengths of each of the six servers and the whole system of a queueing model comprised of five parellel channels centrally linked with a common sixth server by solving steady state equations using generating function technique.

In the present paper, I have studied mathematically and graphically the marginal mean queue length of the whole queueing system. Mean queue length has been studied by taking different values of queueing characteristics taking a fixed value of other parameters.

Mathematical Problem

The whole queueing system consists of six servers S_1, S_2, S_3, S_4, S_5 and S_6 in such a way that S_1 and S_2 which are parallel, connected to S_3 central server and which is further connected to S_4, S_5 and S_6 three parallel servers. Customer comes to S_1 and S_2 from outside with arrival rate λ_1 and λ_2 following the Poission Distribution. Then after getting service s/he may go to S_3 for further service. After getting service from S_3, s/he may go to any of the server S_4, S_5 and S_6 for further service.

Notations

λ_1: Mean Arrival rate at 1st server (S_1)

λ_2 : Mean Arrival rate at 2nd server (S_2)

μ_1: service rate of 1st server (S_1)

μ_2: service rate of 2ndserver.

μ_3: service rate of 3rd server.

μ_4: service rate of 4th server

μ_5: service rate of 5th server

μ_6: service rate of 6th server

P_{ij}: the probability of customer going from ith to jth server.

P_i: the probability of customer going from ith to outside the whole queueing system

$P_{n1,\ n2,n3,n4,n5,n6}$: Probability of having $n1,n2,n3,n4,n5$ and $n6$ customers at S_1, S_2, S_3, S_4, S_5 and S_6 respectively.

If L_q denote the Mean Queue Length of the system.

By Santosh Kumari (2021), the Mean Queue Length of the system is given by:

$$Lq = \frac{-\lambda_1}{\lambda_1-\mu_1} - \frac{\lambda_2}{\lambda_2-\mu_2} + \frac{\lambda_1+\lambda_2-\mu_1-\mu_2}{\mu_1} + \frac{(\lambda_1+\lambda_2)\,p_{34}}{(\mu_4-\mu_3 p_{34})} - \frac{(\lambda_1+\lambda_2)\,p_{35}}{p_{35}\,\mu_3-\mu_5} - \frac{(\lambda_1+\lambda_2)\,p_{36}}{(\mu_3 p_{36}-\mu_6)}$$

Numerical Results and Discussion

- Behaviour of mean queue length (L_q) of the system with respect to probability of customer going from third server to fourth server (p_{34}) for different values of service rate of third server (μ_3) is depicted in Table 1 and in Fig. 1 keeping the values of other parameters as fixed.

Table 1

λ_1=11, λ_2=12, μ_1=15, μ_2=16, μ_4=18, μ_5=19, μ_6=20					
P_{34}	P_{35}	P_{36}	μ_3=17	μ_3=20	μ_3=25
			L_q	L_q	L_q
0.1	0.2	0.7	7.640297077	8.350416667	12.13362519
0.125	0.25	0.625	7.320932871	7.72953149	9.146638655
0.15	0.3	0.55	7.124163942	7.382991453	8.08277193
0.175	0.35	0.475	7.017554317	7.205562397	7.642060749
0.2	0.4	0.4	6.982803217	7.148268398	7.512735043
0.225	0.45	0.325	7.010047904	7.188703704	7.59980604
0.25	0.5	0.25	7.095165945	7.32008547	7.893440956
0.275	0.55	0.175	7.238636199	7.547856061	8.452329802
0.3	0.6	0.1	7.445415266	7.890873016	9.455238095
0.35	0.62	0.03	7.605698426	8.144657919	10.19706681

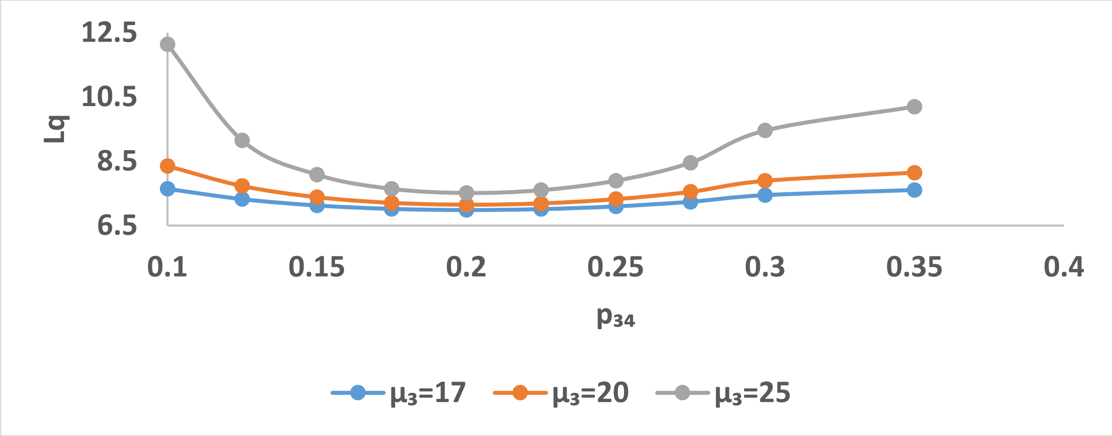

Fig. 1

The following can be interpreted from Table 1 and Fig. 1.

- Mean queue length of the system L_q get decreased with the increase in p_{34} upto p_{34} =0.2 and then increases continuosly.
- Mean queue length of the system L_q increases with the increase in μ_3.

- Behaviour of mean queue length of the system with respect to service rate of first server (μ_1) for different values of service rate of second server (μ_2) is depicted in Table 2 and in Fig. 2 keeping the values of other parameters as fixed.

Table 2

λ_1=10, λ_2=12, μ_3=17, μ_4=18, μ_5=19, μ_6=20, p_{34}=0.6, p_{35}=0.3, p_{36}=0.1			
μ_1	μ_2=16	μ_2=17	μ_2=18
	L_q	L_q	L_q
15	6.687346415	6.020679749	5.554013082
16	6.329013082	5.666513082	5.204013082
17	6.06885902	5.410035491	4.951211962
18	5.870679749	5.215124193	4.759568638
19	5.714247	5.061615421	4.608983842
20	5.587346415	4.937346415	4.487346415
21	5.48215161	4.834532563	4.386913515
22	5.393407021	4.747952476	4.302497931
23	5.31744675	4.673968489	4.230490228
24	5.25163213	4.609965463	4.168298796

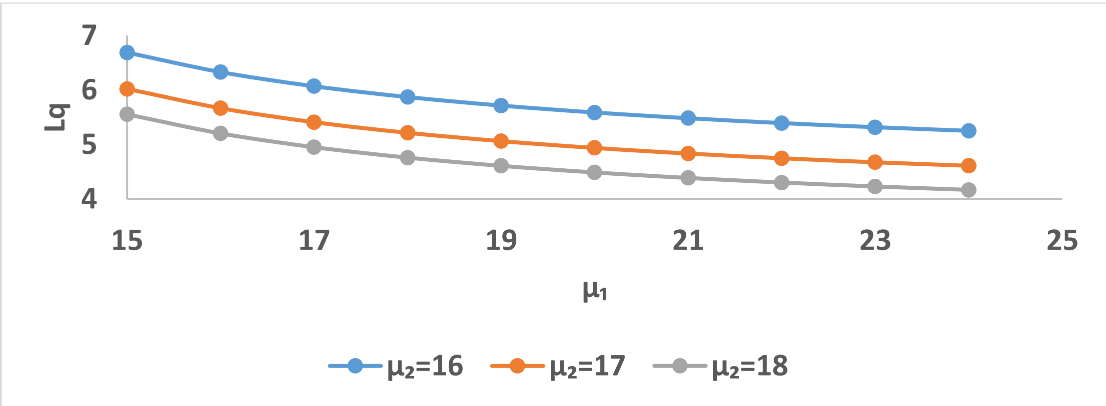

Fig. 2

The following can be interpreted from Table 2 and Fig. 2.

Mean queue length of the system L_q get decreased with the increase in mean service rate of first server (μ_1) as well as service rate of second server (μ_2).

- Behaviour of mean queue length of the system with respect to service rate of third server (μ_3) for different values of service rate of fourth server (μ_4) is depicted in Table 3 and in Fig. 3 keeping the values of other parameters as fixed.

Table 3

λ_1= 10, λ_2=12, μ_1=15, μ_2=16, μ_5=19, μ_6=20, p_{34}=0.6, p_{35}=0.3, p_{36}=0.1			
μ_3	μ_4=18	μ_4=20	μ_4=22
	L_q	L_q	L_q
15	6.440758	6.174091	5.989476
16	6.555783	6.253585	6.04887
17	6.687346	6.341977	6.113683
18	6.839507	6.440956	6.184745
19	7.017788	6.552671	6.263071
20	7.229915	6.679915	6.349915
21	7.487035	6.826374	6.446845
22	7.805854	6.99703	6.555854
23	8.212605	7.198781	6.679504
24	8.750989	7.441465	6.821164

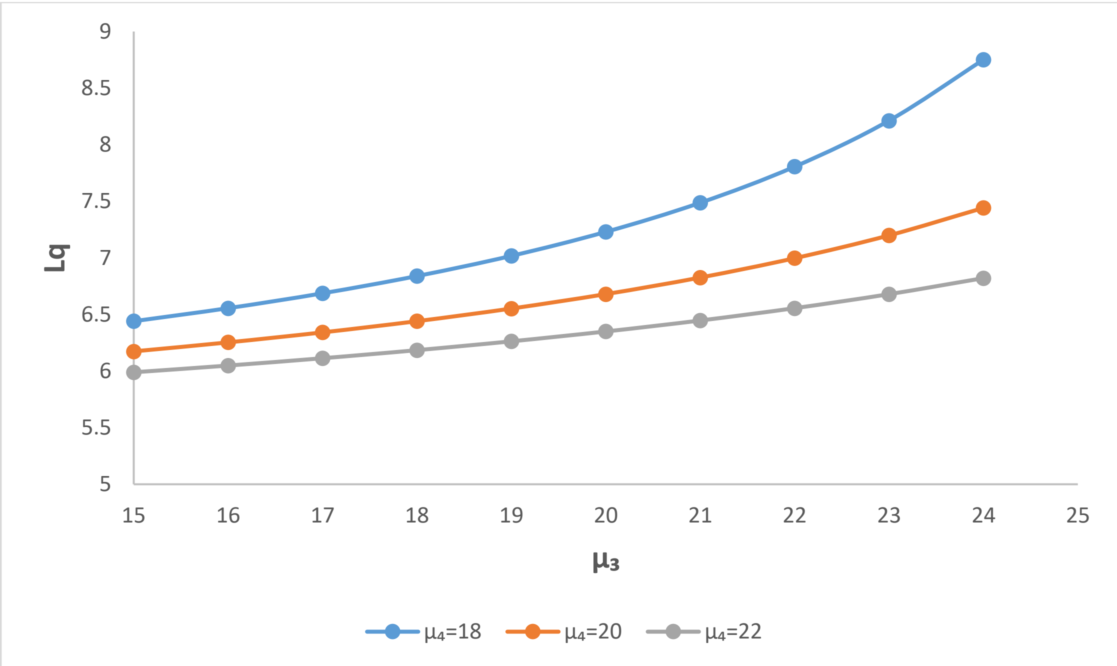

Fig. 3

The following can be interpreted from Table 3 and Fig. 3.

- Mean queue length of the system L_q get increased with the increase in mean service rate of third server (μ_3).
- Mean queue length of the system L_q get decreased with the increase in mean service rate of fourth server (μ_4).

- Behaviour of mean queue length of the system with respect to service rate of third server (μ_3) for different values of service rate of fifth server (μ_5) is depicted in Table 4 and in Fig. 4 keeping the values of other parameters as fixed.

Table 4

λ_1= 10, λ_2=12, μ_1=15, μ_2=16,μ_4=18, μ_6=20, p_{34}=0.6, p_{35}=0.3, p_{36}=0.1			
μ_3	μ_5=18	μ_5=19	μ_5=20
	L_q	L_q	L_q
15	6.657808	6.624091	6.594725
16	6.803349	6.768138	6.73756
17	6.973023	6.936215	6.904348
18	7.173721	7.135205	7.101966
19	7.415275	7.37493	7.340228
20	7.712222	7.669915	7.633651
21	8.087008	8.04259	8.004657
22	8.576227	8.529538	8.489817
23	9.243888	9.194748	9.153111
24	10.21303	10.16125	10.11755

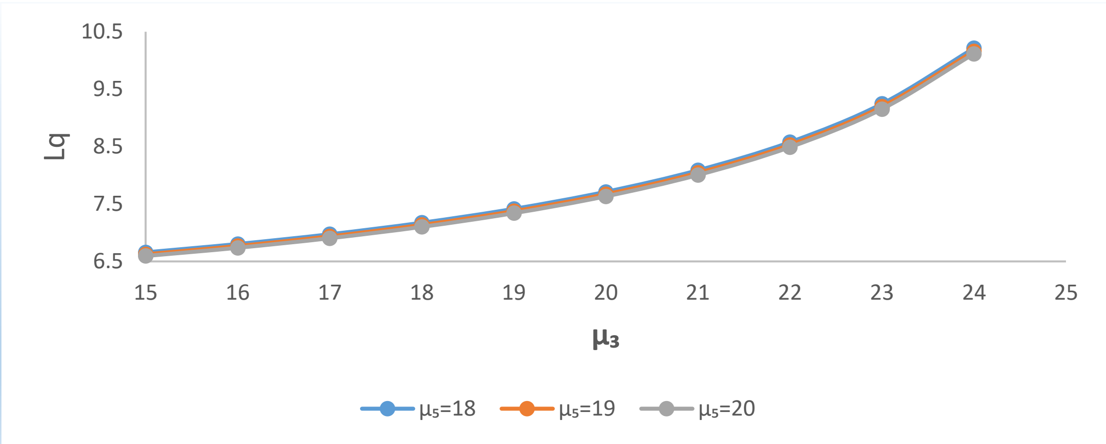

Fig. 4

The following can be interpreted from Table 4 and Fig. 4.

- Mean queue length of the system L_q get increased with the increase in mean service rate of third server (μ_3).
- Mean queue length of the system L_q get decreased with the increase in mean service rate of fourth server (μ_4).

- Behaviour of mean queue length of the system with respect to service rate of third server (μ_3) for different values of service rate of sixth server (μ_6) is depicted in Table 5 and in Fig. 5 keeping the values of other parameters as fixed.

Table 5

λ_1= 10, λ_2=12, μ_1=15, μ_2=16,μ_4=18, μ_5=19, p_{34}=0.6, p_{35}=0.3, p_{36}=0.1			
μ_3	μ_6=10	μ_6=18	μ_6=25
	L_q	L_q	L_q
15	6.580663	6.455172	6.415456
16	6.698122	6.570364	6.530234
17	6.832188	6.702097	6.661548
18	6.98692	6.85443	6.813455
19	7.167846	7.032887	6.991479
20	7.382692	7.245192	7.203344
21	7.64261	7.502494	7.460199
22	7.964309	7.821499	7.778749
23	8.374026	8.228439	8.185228
24	8.915462	8.767014	8.723334

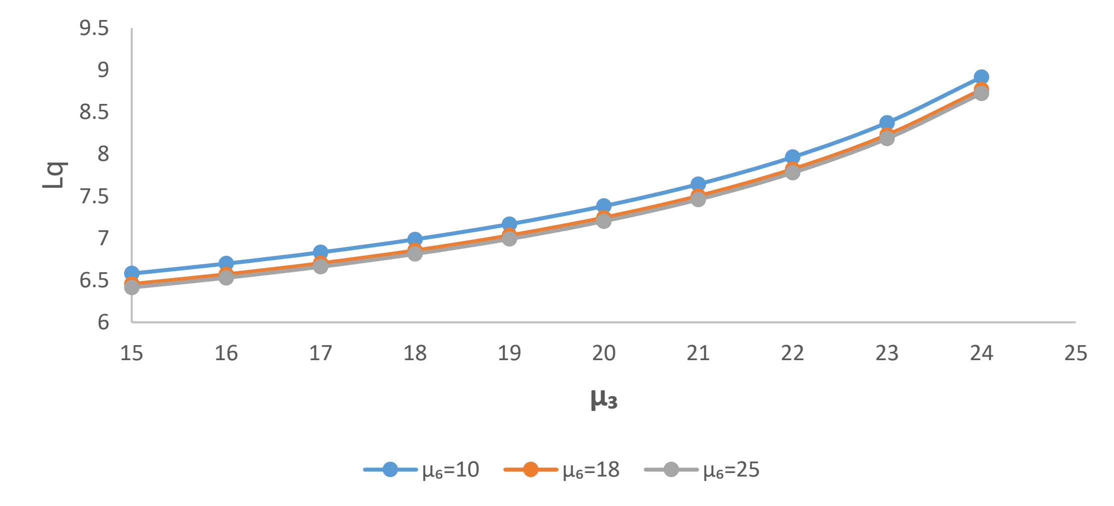

Fig. 5

The following can be interpreted from Table 5 and Fig. 5.

- Mean queue length of the system L_q increases with the increase in **(**μ_3).
- Mean queue length of the system L_q decreases with the increase in **(**μ_6).

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