

Development and Evaluation of a Collaborative Teaching and Learning Model for Mathematics in Secondary Schools

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ABSTRACT

This paper proposes a Collaborative Mathematics Teaching and Learning Model (CMTLM) and describes its major elements using a mathematical framework for evaluation. CMTLM is developed for mathematics classrooms in which students are supposed to produce the right answers to problems and provide explanations, comparisons, justifications, identification of errors, and individual proficiency. The proposed model comprises six phases: diagnostic activation, formation of purposeful groups, collaborative problem solving, mathematical discourse and justification, teacher synthesis, and individual-plus-group assessment. In order to quantify the framework, the paper introduces normalized learning gain, group heterogeneity, participation equity, a weighted collaboration index, and model-effectiveness composite score. Mathematical expressions for treatment effect, Cohen's effect size, analysis of covariance, and reliability are also introduced for future field testing. As an illustration, the quadratic function task is described showing how mathematics can be integrated into the collaborative process. A numerical illustration shows how student pre-test and post-test scores may be transformed into measures of learning gains and collaboration without the use of made-up research results. The design described in this paper is meant to be evaluated through a quasi-experiment using a pre-test post-test design in secondary schools. The paper argues that collaborative teaching of mathematics is researchable if math performance, interaction, equity and accountability are quantitatively measurable variables.

Keywords: Collaborative Learning, Mathematics Education; Secondary School, Mathematical Modelling, Normalized Gain, Cooperative Learning; Mathematical Discourse.

Introduction

Secondary-level mathematics education does not simply involve remembering formulas and performing proven operations. Algebra, geometry, trigonometry, statistics and functions require thinking about relations, symbols, representations, assumptions and proofs. Students may get the same numerical result using various processes, and the comparison of these processes can show deeper conceptual knowledge beyond just the numerical result. That is why, increasingly, mathematics education pays emphasis on explaining, problem-solving, mathematical communication, and justification.

A model of collaborative learning would enable achieving these goals in case the process involved would compel students to articulate their reasoning to fellow learners. It is evident that a learner who justifies that an equation has two roots, two algebraic expressions are equivalent, or a geometrical construction is right is not merely solving a problem but organising mathematical knowledge and making it subject to scrutiny. Unfortunately, this does not necessarily happen through conventional group work

since one learner can dominate, other learners can plagiarise, and the group may concentrate only on the final solution.

Thus, the main goal of this paper is the design of a well-structured Collaborative Mathematics Teaching and Learning Model (CMTLM) together with its mathematical evaluation. As a purely conceptual and methodological study, it will not include any empirical data on classroom performance of participants as none were provided. Instead, equations and indicators are developed that can be used for further empirical analysis of pretest-posttest, observational and participation data. Main research questions that will be addressed are: (1) how to organise collaboration based on the specific features of the discipline; (2) what mathematical indicators could characterise learning and collaboration; and (3) how the model could be in secondary schools?

Theoretical foundation of collaborative mathematics learning

The CMTLM is grounded in theories such as social constructivism, social interdependence theory, cooperative learning, mathematical discourse, and formative assessment. Vygotsky (1978) suggested that learning takes place through socially mediated actions before they become internalised by the learners. With regard to mathematics, social mediation happens when students explain a representation, question a step in a proof, compare solution strategies, or express mathematical ideas in words.

With social interdependence theory, one more design consideration has been added; that is, students need to believe that their objectives are interdependent yet they are personally accountable for learning. Hence, Johnson & Johnson (2009) make a distinction between productive cooperation and non-cooperative collaboration. Among the key components of productive cooperation, there are positive interdependence, individual accountability, promotive interaction, interpersonal skills, and group processing. Gillies (2016) also demonstrates that teacher questions and the quality of peer explanations determine the effectiveness of collaborative efforts.

Research into mathematics specifically also favours structured discourse. The National Council of Teachers of Mathematics (2014) stresses the importance of posing purposeful questions, mathematical discussions, utilizing student thinking and promoting reasoning. Meta-analysis has found positive impact of cooperative learning on maths performance and attitudes (Capar & Tarim, 2015; Kyndt et al., 2013). This does not mean that every lesson needs to be taught through collaboration, but that it needs to be carefully tied in with cognitively challenging mathematics tasks and followed up with individual demonstration of learning.

Development of the CMTLM

The instructional design incorporates a six-step learning process. The purpose of the cycle is to facilitate the interaction between individual cognition and social reasoning and not to replace individual work with constant group work.

Step 1: Diagnostic activation. First, students try to solve a short prerequisite, misconception or retrieval problem individually. The teacher detects previous knowledge and common mistakes. Thus, it helps to set up the lesson and minimize reliance on other people from the very beginning.

Step 2: Deliberate group formation. The students are formed into groups of three to four people. It is important to have enough differences in previous achievements to encourage explanation and avoid any imbalance in a group. Functional roles might include a facilitator, reason checker, scribe and reporter. They change from time to time.

Step 3: Collaborative problem solving. Each group is assigned a problem that includes interpretation, comparison, generalization or representation. The problem cannot be separated into unconnected subtasks. It is necessary to find a joint mathematical argument or solution.

Stage 4: Mathematical arguments and justification. Groups share approaches, probe each other, uncover assumptions, and see whether various algorithms produce the same results. The teacher chooses two approaches and challenges students to justify their arguments using definitions, mathematical operations, sketches, or numbers.

Stage 5: Teacher-mediated synthesis. The teacher ties student approaches to formal mathematical notation, clears up any remaining misconceptions, and draws out key ideas. Direct instruction is preserved, but is based on insights obtained during student problem solving.

Stage 6: Assessment of individual and group work. Students take an individual quiz or test and reflect on the group's performance. The model thus assesses whether social reasoning has translated into individual knowledge.

Stage	Main mathematical action	Evidence produced
1	Individual prerequisite reasoning	Baseline score and misconceptions
2	Balanced group formation	Group composition data
3	Shared problem solving	Written solution and strategy record
4	Explanation and justification	Discourse/participation evidence
5	Formalisation	Correct mathematical statement
6	Individual assessment	Post-learning score and reflection

Mathematical Formulation of the Model

The evaluation of a mathematical education model is made easier by expressing the central constructs of the model in observable terms. If the student is in the group 'g', then let $P(i)$ be the pretest score of the student, $Q(i)$ be the posttest score, and M be the maximum score.

- Individual and group learning gain**

A simple raw gain is $Q(i) - P(i)$, but students with different pretest scores do not have equal room for improvement. The normalized gain is therefore defined as:

$$g_i = \frac{Q_i - P_i}{M - P_i}, \quad P_i < M$$

Here, $g(i)$ represents the proportion of the student's available improvement that was achieved. Let $G(g)$ denote the mean normalized gain for a group containing $n(g)$ students:

$$G_g = \frac{1}{n_g} \sum_{i=1}^{n_g} g_i$$

Let G denote the class-level mean normalized gain across all N students:

$$G = \frac{1}{N} \sum_{i=1}^N g_i$$

These indicators should be interpreted with actual sample distributions and not used as substitutes for formal statistical testing.

- Group heterogeneity and balance**

Since deliberate grouping falls under CMTLM, heterogeneity in terms of prior achievement within each group can be calculated. The heterogeneity within group g is defined as follows:

$$H_g = \sqrt{\frac{1}{n_g} \sum_{i=1}^{n_g} (P_i - P_g)^2}$$

Greater value of $H(g)$ implies greater heterogeneity in prior achievement within each group. However, it does not imply any differences in strength between two groups. Suppose $P(c)$ is the mean achievement of a class on the pretest, and $s(P)$ is its standard deviation. The group-balance index is:

$$B_g = \frac{1}{1 + |P_g - P_c|/s_P}$$

Greater values of $B(g)$ closer to 1 imply that the mean prior achievement of the group is close to the mean of the class.

- Participation equity**

Let $t(i,g)$ denote the total number of significant mathematical contributions by student i within group g . We define the mean and standard deviation of the contribution numbers to be $m(g)$ and $s(t,g)$, respectively. The coefficient of variation is:

$$CV_g = \frac{S_{t,g}}{m_g}$$

Participation equity is then transformed to a bounded index:

$$E_g = \frac{1}{1 + CV_g}$$

If contributions are evenly distributed, $CV(g)$ is small and $E(g)$ approaches 1. If one or two students dominate, the dispersion increases and $E(g)$ decreases.

- **Collaboration index**

Collaboration quality can be represented by four rubric-based dimensions: mathematical explanation $A(g)$, justification $J(g)$, interdependence $I(g)$, and participation equity $E(g)$. Each dimension is scaled from 0 to 1. A weighted collaboration index is:

$$C_g = w_A A_g + w_J J_g + w_I I_g + w_E E_g$$

subject to

$$w_A + w_J + w_I + w_E = 1, \quad w_k \geq 0$$

Equal weights may be used initially, but validated weights could later be estimated through expert judgement, factor analysis, or predictive modelling.

- **Composite model-effectiveness score**

CMTLM is intended to produce both mathematical learning and productive collaboration. For descriptive classroom monitoring, a composite score can combine the mean group learning gain $G(g)$ and collaboration quality:

$$S_g = \lambda G_g + (1 - \lambda) C_g, \quad 0 \leq \lambda \leq 1$$

The parameter λ determines the emphasis placed on academic learning. For example, $\lambda = 0.60$ gives 60% weight to normalized learning gain and 40% to collaboration quality. The score is a monitoring index, not an inferential estimate of causal effectiveness.

- **Statistical evaluation of treatment effects**

In a field trial, the effect of CMTLM should be tested against a comparison condition. A difference-in-differences estimate can be written as:

$$\Delta = (Q_T - P_T) - (Q_C - P_C)$$

where $P(T)$, $Q(T)$, $P(C)$, and $Q(C)$ denote the corresponding treatment and comparison sample means. Standardised effect size may be reported using Cohen's d :

$$d = \frac{Q_T - Q_C}{s_p}$$

with pooled standard deviation

$$s_p = \sqrt{\frac{(n_T - 1)s_T^2 + (n_C - 1)s_C^2}{n_T + n_C - 2}}$$

To adjust posttest achievement for baseline differences, an ANCOVA model can be specified as:

$$Q_i = \beta_0 + \beta_1 T_i + \beta_2 P_i + \varepsilon_i$$

where $T(i) = 1$ for CMTLM students and $T(i) = 0$ for comparison students. The coefficient β_1 estimates the adjusted difference in posttest performance, subject to the assumptions of the model.

If a multi-item collaboration scale is used, internal consistency can be checked using Cronbach's alpha:

$$\alpha = \frac{k}{k-1} \left(1 - \frac{\sum_{j=1}^k \sigma_j^2}{\sigma_X^2} \right)$$

where k is the number of items, $\sigma(j)^2$ is the variance of item j , and $\sigma(X)^2$ is the variance of the total score.

Illustrative mathematics task within the model

A quadratic-function task demonstrates how disciplinary mathematics can be incorporated into CMTLM. Students are given

$$f(x) = x^2 - 6x + 5$$

and asked to represent the same function in factored and vertex forms. One group may factor:

$$f(x) = (x - 1)(x - 5)$$

which shows that the zeros are $x = 1$ and $x = 5$. Another group may complete the square:

$$f(x) = x^2 - 6x + 9 - 4 = (x - 3)^2 - 4$$

which shows that the vertex is $(3, -4)$. Students then explain why all three expressions represent the same parabola and connect each form to different information about the graph. An equivalence check could work through the expansion of $(x - 1)(x - 5)$, whereas another student would be able to give a reason about the axis of symmetry $x = 3$.

This is a good approach since cooperation in the project would be based on the relationships in mathematics and not on the division of routine assignments. An individual question to test the exit of the student from the class could include the transformation of a quadratic expression, for example $x^2 - 8x + 12$, without any help from peers.

Numerical example (calculation)

The following numbers serve an illustrative purpose only and are presented to prove the above-presented formulas; the data cannot be considered scientific. Assume that four students have the maximum possible score $M = 100$.

Student	Pretest P(i)	Posttest Q(i)	Normalized Gain g(i)
A	50	75	0.500
B	60	78	0.450
C	40	70	0.500
D	70	84	0.467

The group mean normalized gain, denoted G_g , is approximately:

$$G_g = \frac{0.500 + 0.450 + 0.500 + 0.467}{4} = 0.479$$

Assume meaningful contribution counts are 9, 8, 7, and 8. Their mean is 8 and population standard deviation is approximately 0.707. Thus,

$$CV_g = \frac{0.707}{8} = 0.088, \quad E_g = \frac{1}{1 + 0.088} = 0.919$$

If rubric scores are $A(g) = 0.85$, $J(g) = 0.80$, $I(g) = 0.90$, and $E(g) = 0.919$, with equal weights, then:

$$C_g = 0.25(0.85 + 0.80 + 0.90 + 0.919) = 0.867$$

Using $\lambda = 0.60$:

$$S_g = 0.60(0.479) + 0.40(0.867) = 0.634$$

This value should be interpreted only as a composite monitoring score based on the chosen scaling and weights. It does not mean that the model has been proven to be 63.4% effective.

Proposed methodology for empirical evaluation

A future evaluation should use a quasi-experimental pretest-posttest design involving several secondary-school mathematics classes. Existing classes may be assigned to CMTLM or comparison instruction when individual randomisation is impractical. An intervention of approximately eight to twelve weeks would allow students to learn the collaboration routines and apply them across several mathematics topics.

The primary dependent variable should be achievement on a curriculum-aligned mathematics test containing procedural, conceptual, and non-routine problem-solving items. Some secondary variables might include mathematical reasoning, attitude toward mathematics, equity of participation, and quality of collaboration. Pretest would provide baseline variable $P(i)$, whereas posttest would give $Q(i)$.

Classroom observation would capture the data on explanations, justifications, interdependencies, and contributions necessary for C(g).

Measuring implementation fidelity is another important issue to consider. Observation checklist would rate lessons' compliance with independent activation, grouping purposes, cognitively challenging activities, student explanations, public justifications, teacher synthesizing, and individual assessment. In absence of fidelity evidence, negative results might indicate flaws in implementation rather than flaws in the theory.

Descriptive statistics should be followed by application of ANCOVA or multilevel modelling, where students are nested in either classrooms or schools. Effects of treatment need to be reported using confidence intervals and effect sizes, and not p-values alone. Multi-item instruments need reliability evaluation, and further subgroup analysis might reveal differences in effects among students with different levels of prior attainment. Ethical considerations might involve obtaining school approval, consent or assent, confidentiality, and equality of curriculum access.

The principal null hypothesis can be expressed as:

$$H_0: \mu_T = \mu_C$$

against the directional alternative:

$$H_1: \mu_T > \mu_C$$

where μ_T and μ_C represent adjusted posttest achievement means for CMTLM and comparison students respectively.

Discussion

The new model corrects one of the key shortcomings of collaborative mathematics instruction which is the lack of a mathematical way of measuring collaboration. In CMTLM model, the classroom processes are related to specific indicators such as g(i) which stands for learning gain, H(g) and B(g) for group composition, E(g) for participation equity and C(g) for collaboration quality. It allows researchers to look into whether there is a correlation between achievement change and the quality and distribution of mathematical interactions.

Formalising the model does not make it objective per se. First of all, the observation scores have to be based on the correct rubrics, the contribution counts should reflect the amount of participation and not the amount of talking and finally, the composite weights have to be justified. The key benefit of formalisation is thus its transparency.

CMTLM also maintains an essential balance between group reasoning and individual accountability, because collaborative activity is employed in generating explanations and comparing strategies, but the individual exit task is used in determining whether the learner can do the mathematics alone. The model is therefore compatible with secondary school assessment practices where the final achievement is always assessed individually.

Conclusion

This paper presented a Collaborative Mathematics Teaching and Learning Model (CMTLM) for secondary schools and enhanced its strength with an explicit mathematical evaluation of the model. The model comprises six steps, which include diagnostic activation, purposeful grouping, collaborative problem solving, mathematical discourse, teacher synthesis, and individual and group assessment. The mathematical representation of CMTLM incorporates normalized learning gain, group heterogeneity, group balance, participation equity, weighted collaboration index, and overall monitoring score. Difference-in-differences, Cohen's d, ANCOVA, reliability, and hypothesis test equations were mathematically described in order to conduct future empirical research.

No experimental evidence for the effectiveness of the model is claimed in this paper since there was no classroom dataset used. Instead, this paper presents a researchable framework that can be used for any secondary school data. By incorporating actual mathematics problems along with quantitative measures of learning and collaboration, CMTLM gives clear grounds for evaluating the impact of collaborative teaching on mathematics performance.

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