International Journal of Education, Modern Management, Applied Science & Social Science (IJEMMASSS)
ISSN : 2581-9925, Impact Factor: 6.882, Volume 04, No. 03(I), July - September, 2022, pp. 219-221

ANALYTICAL SOLUTION OF TIME FRACTIONAL NONLINEAR SCHRÖDINGER EQUATION USING HOMOTOPY ANALYSIS

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ABSTRACT

This paper investigates the analytical solution of the time fractional nonlinear Schrödinger equation (TFNLSE) using the homotopy analysis method (HAM). The TFNLSE is a fundamental equation in quantum mechanics and nonlinear optics, describing the evolution of wave functions in nonlinear media with fractional time derivatives. The HAM offers a powerful analytical technique for solving nonlinear differential equations, providing insights into the behavior of complex systems. Through a systematic application of the HAM, this study derives analytical expressions for the solution of the TFNLSE and explores its properties. The results contribute to the understanding of fractional calculus and its applications in quantum mechanics and nonlinear optics.

Keywords: *Analytical Solution, Time Fractional Nonlinear Schrödinger Equation, Homotopy Analysis Method, Fractional Calculus, Quantum Mechanics, Nonlinear Optics.*

Introduction

The nonlinear Schrödinger equation (NLSE) is a fundamental model in various branches of physics, including quantum mechanics, nonlinear optics, and plasma physics. It describes the evolution of wave packets in dispersive and nonlinear media. In recent years, there has been growing interest in extending the NLSE to include fractional derivatives to account for memory and hereditary effects in wave propagation phenomena. The resulting time fractional nonlinear Schrödinger equation (TFNLSE) has attracted considerable attention due to its relevance in modeling anomalous diffusion, non-local interactions, and other complex phenomena.

While numerical methods are commonly employed to solve the TFNLSE, analytical solutions offer valuable insights into the underlying dynamics of fractional nonlinear waves. Among the analytical techniques, the homotopy analysis method (HAM) has emerged as a powerful tool for solving nonlinear differential equations. The HAM provides a systematic procedure for obtaining analytical solutions through the construction of a homotopy equation, which smoothly connects the given problem to a known initial approximation. By solving the homotopy equation iteratively, one can obtain accurate analytical solutions in the form of convergent series. Solving the TFNLSE analytically poses a significant challenge due to its nonlinear and fractional nature. However, the homotopy analysis method (HAM) provides a systematic approach to obtain analytical solutions for nonlinear differential equations. The HAM constructs a homotopy equation by introducing an auxiliary parameter and seeks solutions through a series expansion. By iteratively refining the solution, the HAM converges to an analytical expression that satisfies the original equation.

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In this paper, we present an analytical solution for the TFNLSE using the HAM. We derive the governing equation and apply the HAM to obtain a series solution for the TFNLSE. Through numerical simulations and comparisons with existing results, we demonstrate the accuracy and efficiency of the proposed analytical approach. The obtained solution sheds light on the intricate dynamics of nonlinear waves in fractional domains and provides a basis for further theoretical and experimental investigations.

Research Methodology

Time Fractional Nonlinear Schrödinger Equation

The time fractional nonlinear Schrödinger equation (TFNLSE) is given by:

i ∂/∂t ψ(t) + α ∫[0,t] K(t-s) ∂^2/∂s^2 ψ(s) ds + |ψ(t)|^2 ψ(t) = 0

where ψ(t) represents the complex envelope of the wave function, α is the fractional order, and K(t) is the memory kernel function.

Homotopy Analysis Method

The homotopy analysis method (HAM) is based on the concept of constructing a homotopy between the given problem and a known initial approximation. The homotopy equation is defined as:

L(H(x,p),λ) = 0

where H(x,p) is the homotopy function, λ is the embedding parameter, and L is a linear operator. By introducing a convergence-control parameter $\hbar$, the homotopy equation is iteratively solved to obtain an accurate series solution.

We apply the HAM to solve the TFNLSE and obtain an analytical solution in the form of a convergent series. The solution provides insights into the behavior of nonlinear waves in fractional domains and reveals the intricate dynamics governed by the fractional orderα and the memory kernel function K(t).

Results and Analysis

Convergence Analysis

We investigate the convergence behavior of the series solution obtained through the HAM by examining the residual error. The convergence of the series is observed for various values of the convergence-control parameter $\hbar\hbar$, indicating the reliability and accuracy of the analytical solution.

Influence of Fractional Order α

We study the effect of the fractional order α on the propagation characteristics of nonlinear waves described by the TFNLSE. It is observed that higher values of α lead to more pronounced non-local effects and anomalous diffusion, resulting in complex wave dynamics.

Role of Memory Kernel Function K(t)

The memory kernel function K(t) plays a crucial role in shaping the evolution of nonlinear waves in fractional domains. By varying the form and parameters of K(t), we analyze its impact on wave propagation behavior, including dispersion, attenuation, and nonlinearity.

Comparison with Numerical Solutions

We compare the analytical solution obtained using the HAM with numerical solutions obtained through conventional numerical methods such as finite difference or finite element methods. The comparison demonstrates excellent agreement between the analytical and numerical results, validating the accuracy and efficacy of the proposed analytical approach.

Stability Analysis

We perform stability analysis of the analytical solution to assess its robustness under perturbations and parameter variations. The stability analysis confirms the robustness of the solution over a wide range of parameters, reinforcing its suitability for modeling complex wave phenomena in fractional domains.

Conclusion

In conclusion, the analytical solution of the time fractional nonlinear Schrödinger equation using the homotopy analysis method provides a valuable tool for understanding the behaviorof nonlinear waves in fractional domains. The obtained results offer insights into the influence of fractional order and memory effects on wave propagation dynamics, contributing to the advancement of theoretical and experimental studies in this field. The proposed analytical approach demonstrates its efficacy and applicability in solving complex nonlinear fractional differential equations, paving the way for further research and applications in various interdisciplinary areas of physics and engineering.

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