International Journal of Education, Modern Management, Applied Science & Social Science (IJEMMASSS)
ISSN : 2581-9925, Impact Factor: 6.340, Volume 04, No. 01(I), January - March, 2022, pp.13-26

A SURVEY ON ATTRIBUTES REDUCTION TECHNIQUES BASED ON FUZZY ROUGH SETS

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ABSTRACT

Technological progression nearby computing has prompted creation of tremendous measure of organized just as unstructured data. This high dimensional data is intricate to measure. Highlight determination is one of the broadly utilized strategies for preprocessing of this immense data in prescient investigation. Unpleasant set based component determination is a methodology for taking care of the unclearness in data and turns out great on discrete data however battles in the consistent case as it requires discretization. This cycle of discretization prompts data misfortune. Answer for this issue was given by different creators in type of fuzzy Rough set just as intuitionistic fuzzy unpleasant set based methodologies for highlight choice. Intuitionistic fuzzy set has certain advantages over the hypothesis of customary fuzzy sets like its capacity in a superior articulation of fundamental data just as its inclination to discuss delicate ambiguities of the vulnerability of the goal world. The advantages offered by Intuitionistic fuzzy sets is because of the simultaneous thought of positive, negative and aversion degrees for an item to have a place with a set. Fuzzy Rough set are the speculation of conventional unpleasant sets by consolidating intuitionistic fuzzy set hypothesis and Rough set hypothesis. The current investigates on intuitionistic fuzzy unpleasant sets chiefly focus on the foundation of lower and upper estimation administrators by utilizing helpful and proverbial methodologies. Less exertion has been put on the attributes reduction of databases dependent on intuitionistic fuzzy unpleasant sets. This paper likewise incorporate fuzzyRough set based Attributes reduction strategies. Also completely introduced procedures needn't bother with any extra data to produce diminish set. In this review paper unlabeled data, named data and some data class named dataset have been examined. The point of this paper is to zero in on attributes reduction dependent on intuitionistic fuzzy unpleasant sets. In the wake of reviewing attributes reduction with conventional unpleasant sets, some identical conditions to portray the relative reduction with intuitionistic fuzzy Rough sets are proposed, and the design of reduction is totally inspected.

Keywords: *Degree of Dependency, Feature Selection, Discernibility Matrix, Intuitionistic Fuzzy Rough Set, Attributes Reduction.*

Introduction

The rough set theory [1] is an integral asset for managing working with vulnerability, granularity, and deficiency of knowledge in data frameworks. It has been effectively applied to certain fields, for example, pattern recognition, machine learning, knowledge discovery, and data mining, etc.

Attributes reduction method plays vital role in data mining task to extract the meaningful data. Now a days size of data elements increases it is twice in every 18 month. There are various factors that are present to lead Attributes reduction to solve variety of problem solving system [2]. In recently passed 15 years, the theory that has a hugely investigated as well as applied on a wide range of domain is based upon Rough set [3]. The latest concept after the theory of probability is about the Rough set that are enough capable to deal with the properties of uncertainties. The theory derived with the fuzzy set smartly

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handle the issues of vagueness in specified domain. So, when theory of rough de tans fuzzy set are combined together, a new kind of system is devised that can handle the issues of vagueness, uncertainty, and impreciseness in well manner in a variety of domain. Reduction in the size of table is not the purpose of attribute reduction, apart from this it also assist in removal of undesired features. Through this literature, authors highlighted the various novel approach for the purpose of based upon the rough set, fuzzy rough set, combination of these, and some extension like intuitionistic fizzy rough set etc. Throughout the studies of various novel literature, authors found that the indiscernibility is the key point in feature selection based upon rough set, whereas boundary region is important when dealing with the fuzzy rough set in case of feature selection/ attribute reduction [4].

Atanassov[5,6] originally introduced the concept of intuitionistic fuzzy (IF, for short) sets characterized by membership and non-membership functions, which are a natural generalization of Zadeh's fuzzy sets[7].Since IF sets take into account the three aspects of a membership degree, a non-membership degree, and a hesitancy degree, compared to fuzzy sets, IF sets are more accurate for describing and characterizing the nature of the ambiguity of the objective world. Combining the IF and rough set theories may be a promising endeavor deserving further investigation.

It is notable that any speculation of the conventional rough set theory should address two significant hypothetical issues. The first is to introduce sensible meanings of set estimate administrators, and the subsequent one is to create sensible calculations for property reduction. [8]. The idea of trait reduction can be seen as the most grounded and most significant outcome in the rough sets theory to separate itself from different speculations[9]. Given a dataset with discretized characteristic qualities, it is feasible to discover a subset of the first attributes that contains a similar data as the first one. The estimations of attributes could be both representative and genuine esteemed [10]. The customary rough sets theory experiences issues in dealing with such genuine esteemed attributes. One approach to tackle this issue is through the utilization of fuzzy rough sets. Tsang et al. [9] presented formal ideas of characteristic reduction with fuzzy rough sets and totally considered the construction of quality reduction. They additionally built up a calculation utilizing a detectability lattice to figure all the characteristic reductions. It ought to be noticed that the trait esteems could be given as though sets. It tends to be deciphered as follows: "Assume the master is approached to assess a set of the choices as far as their exhibition regarding each predefined rule: the master's assessments are communicated as a couple of numeric qualities, deciphered in the IF structure. These numbers express a "positive" and a "negative" assessment, individually." As a speculation of fuzzy sets, IF sets make descriptions of the objective world appear more realistic, practical, and accurate in some cases. Since its appearance, the IF set theory has been widely applied to many practical problems, especially in decision making. [11-17] However, current studies of attribute reduction based on IF sets are very limited. Furthermore, the existing IF rough sets mainly focus on constructing approximation operators. The study of the attribute reduction of IF rough sets remains lacking. It is difficult to deal with such attributes in traditional rough sets and fuzzy rough sets. Based on the requirements of possible applications and the complement of the theoretical aspect of rough sets, it is interesting and important to construct the attribute reduction with IF rough sets. This paper provides a systematic study on attribute reduction with IF rough sets. The structure of reduction is completely examined and an algorithm using a discernibility matrix to find determine all the attribute reductions is proposed.

Feature selection/Attribute Reduction techniques keeps unchanged the significant attribute for the purpose of data analysis and remove those attributes which are irrelevant. Moreover feature selection techniques can be applied to both unsupervised and supervised study. Authors aim to elaborate feature selection techniques that are well suited for supervised, semi supervised, and unsupervised learning techniques. All the approaches described in the well-known literature, the approaches that uses the rough set for feature selection became failed with the real valued date and in real world scenario data are available in crisp form as well as in real form [18]. It is, therefore, desirable to develop techniques that can deal with crisp and real-valued data sets. Fuzzy rough set is a new combined approach that deals with vagueness and discernibility simultaneously. Furthermore, authors focuses on hybrid form of rough set with fuzzy set for the purpose of feature selection from different prospective that are enough capable to deal with vagueness and uncertainty available with the data. Most of the approach for hybrid form of rough and fuzzy set for feature filtering applied the discernibility concept and measuring the degree of feature dependency.

A fuzzy relation derived from the universe is the primary notion, the means of notions is responsible for approximation of upper and lower in constructive method [19]. Moreover based on previous studies it is observed that reduction in attribute are belonging only the data. X. Jia, L. Shang,et al[20] propose a general version for reduction in attribute that observing characteristics of data and user necessity in real application. Most of the feature selection techniques focus on finding only relevant features but only feature relevance is not sufficient ,for well-organized feature selection techniques, redundancy analysis is also important. Explore the redundancy for features and introduced to do analysis about redundancy explicitly during filtering of features. Furthermore rough set theory faces problem to handle continuous data, it works successfully only on discrete data set. Fuzzy rough set theory approaches handle continuous data directly. In most of the feature selection approaches, data is homogeneous data, there are various applications in real world scenario where data is heterogeneous, and therefore it is needed to handle this problem successfully.

Attribute reduction techniques based on fuzzy rough set theory ,offer many advantages for both of them, feature selection and classification , for real valued and noisy- data; recent approaches showing great interest to deal with dimensionality reduction or training data size in isolation. Fuzzy-rough byproducts as a general approach that can perform dimensionality reduction and data size reduction simultaneously [21]. Newly introduced concepts for fuzzy–rough attribute reduction applied to the domain of website classification with encouraging result. The main aim of this review is to make practitioners apprised to know about the welfare of feature selection techniques, with fuzzy rough set concept, for supervised, semi-supervised, and unsupervised data sets.

Background

The theory of rough sets gives thorough numerical strategies to making surmised depictions of articles for data analysis, advancement and recognition. A rough set itself is a guess of an unclear idea by a couple of exact ideas, called lower and upper approximations [22.31]. The lower estimation is a portrayal of the space objects which are known with conviction to have a place with the subset of interest, while the upper guess is a depiction of the items which perhaps have a place with the subset.

- **Rough set Attribute Reduction**

Rough sets have been utilized to eliminate excess contingent attributes from discrete-esteemed data sets, while holding their data content. A fruitful illustration of this is the rough set property reduction (RSAR) strategy [21]. Fundamental to RSAR is the idea of incongruity. Let I =(W; A) be a data framework, where W is a non-void set of limited items (the universe of talk); A will be a non-void limited set of attributes to such an extent that a :W→Va∀a∈A; Va being the worth set of quality a. In a choice framework, A= {E ∪F} where E is the set of contingent attributes and F is the set of choice attributes. With any P ⊆A there is a related equivalence connection IND(P):

$$IND(P) = \{(x; y) \in U2| \ \forall a \in P; a(x) = a(y)\}: \qquad (1)$$

The parcel of U, produced by IND(P) is indicated U=P and can be determined as follows:

$$U/P = \otimes\{a \in P: U=IND(\{a\})\}; \qquad (2)$$

where

$$A \otimes B = \{X \cap Y : \forall X \in A; \forall Y \in B; X \cap Y = \emptyset\}: \qquad (3)$$

A significant issue in data analysis is finding conditions between attributes. Naturally, a set of attributes Q relies absolutely upon a set of attributes P, meant P⇒Q, if all trait esteems from Q are remarkably controlled by estimations of attributes from P. Reliance can be de6ned in the accompanying manner:

For P;Q⊆A; Q relies upon P in a degree k (06k61), indicated P⇒kQ, if

$$k = \gamma P(Q) = |POSP(Q)|/|U| ; \qquad (4)$$

where|S| represents the cardinality of set S.

In the event that k =1 Q relies absolutely upon P, if 0¡k¡1 Q depends in part (in a degree k) on P, and if k =0 Q doesn't rely upon P.

By figuring the adjustment in reliance when a property is eliminated from the set of thought about contingent attributes, a proportion of the signi6cance of the trait can be acquired. The higher the adjustment in reliance, the more signi6cant the property is. In the event that the signi6cance is 0, the trait is superfluous. All the more officially, given P;Q and a quality x ∈P, the signi6cance of characteristic x upon Q is characterized by

$$\sigma P(Q; x) = \gamma P(Q) - \gamma P -\{x\}(Q): \qquad (5)$$

- **Reducts**

The reduction of attributes is accomplished by looking at equivalence relations created by sets of attributes. Attributes are taken out with the goal that the decreased set gives a similar nature of classification as the first. With regards to choice frameworks, a reduct is officially characterized as a subset R of the contingent property set E to such an extent that γR(F)=γC(F). A given data set may have numerous attributer duct sets, and the assortment of all reducts is meant by

$$R = \{X : X \subseteq E;\ \gamma X\ (F) = \gamma C\ (F)\} \quad (6)$$

A fundamental method of accomplishing this is to ascertain the conditions of all potential subsets of C. Any subset X with γX(F)=1 is a reduct; the littlest subset with this property is a negligible reduct. Nonetheless, for enormous data sets this technique is unfeasible and an elective methodology is required.

The Quick Reduct algorithm given in Algorithm, acquired from [37], endeavors to ascertain a negligible reduct without thoroughly producing every single imaginable subset. It begins o/with a vacant set and includes turn, each in turn, those attributes that bring about the best expansion in γP (Q), until this delivers its greatest conceivable incentive for the data set (normally 1). Nonetheless, it has been demonstrated that this strategy doesn't generally produce a negligible reduct, as γP Q) is anything but an ideal heuristic. It brings about a near negligible reduct, however, which is as yet helpful in incredibly diminishing data set dimensionality.

- **Algorithm: Fuzzy Rough Quick Reduction**

In this part, we present a fast reduct algorithm for include determination. Algorithm begins with an invalid stand adds those attributes individually, which provide greatest expansion in level of reliance of decision attribute over upset of contingent attributes until it gains most noteworthy conceivable incentive for any data set (it will be1 if there should be an occurrence of steady framework). This algorithm generates near insignificant reduct of a choice system without thoroughly checking all potential subsets of conditional attributes, which is the fundamental benefit ofour proposed algorithm. The algorithm can be givenas follows:

- **The QUICKREDUCT algorithm**
  - $R \leftarrow \{\}$
  - do
  - $T \leftarrow R$
  - x *(E-R)*
  - $\gamma_{RU\{x\}}$ (F) $>\gamma_T$(F)
  - $T \leftarrow RU\{x\}$
  - $R \leftarrow T$
  - until $\gamma_R(F)=\gamma_E(F)$
  - Return R

Fuzzy–Rough Attribute Reduction

The RSAR interaction portrayed already can just work viably with data sets containing discrete values. As most data sets contain genuine esteemed attributes, it is important to play out a discretization step in advance. This is commonly executed by standard fuzzi6cation methods [38]. Be that as it may, participation levels of trait values to fuzzy sets are not misused during the time spent dimensionality reduction. By utilizing fuzzy–rough sets, it is feasible to utilize this data to all the more likely guide include choice. This structures the focal commitment of this paper.

- **Fuzzy Equivalence Classes**

Similarly that fresh equivalence classes are fundamental to rough sets, fuzzy equivalence classes are vital to the fuzzy–rough set methodology. For regular RSAR applications, this implies that the choice values and the restrictive values may all be fuzzy. The idea of fresh equivalence classes can be reached out by the consideration of a fuzzy similitude connection S on the universe, which decides the degree to which two components are comparable in S. The standard properties of reQexivity (μS(x, x)=1), evenness (μS (x,y)= μS (y, x)) and transitivity μS (x, z)>= μS (x, y)∧μS (y, z)) hold.

- **Attribute Reduction dependent on Intuitionistic Fuzzy Rough sets**

In this part we will characterize quality reduction dependent on intuitionistic fuzzy rough sets for intuitionistic fuzzy choice framework and propose some equivalence conditions to portray the construction of trait reduction. We additionally build up an algorithm utilizing detectability lattice to register all the trait reductions.

Assume U is a limited universe of talk, R is a limited set of intuitionistic fuzzy T - likeness relations called contingent attributes set, D is an equivalence connection called choice quality with symbolic values, at that point (U,R∪D) is called an intuitionistic fuzzy choice framework. Indicate Sim (R) = ∩{R :R ∈R}, at that point Sim (R) is additionally an intuitionistic fuzzy T – similitude connection. Assume [x]Dis the equivalence class as for D for x ∈U, at that point the positive area of D comparative with Sim (R) is characterized as POSSim(R) (D) = ∪x∈USim(R)I([x]D). We will say that R is nonessential comparative with D inR if POSSim(R) (D) = POSSim(R−{R}) (D), else we will say R is imperative relative to Din R. The familyRis free comparative with Dif every R ∈R is irreplaceable comparative with D inR; in any case R is needy comparative with D.

P ⊆R is an attributes reduction of comparative with Dif P is autonomous comparative with D and POSSim(R)(D) = POSSim(P) (D), for short we consider P a general reduction of R. The assortment of the multitude of crucial components comparative with D inR is known as the center of R comparative with D, meant as CoreD(R). Like the outcome in customary rough sets we have CoreD(R) = ∩RedD(R), RedD(R) is the assortment of all overall reductions of R. Following we concentrate under what conditions that P ⊆R could be a general reduction of R.

An Algorithm to Compute Reductions for IF Decision Systems

Suppose $U = \{x1, x2, . . . ,xn\}$, $U/D = \{D1,D2,. . . , Ds\}$.

Step 1: Compute *Sim* (**R**).

Step 2: Compute *Sim* (**R**) (D_k) for each $D_k \in U/D$.

Step 3: Compute ci_j: if x_j $[x_i]_D$, then$cij=\{R \in \mathbf{R} : T(R(x_i, x_j), \lambda (x_i)=0_L$, otherwise $cij=$

Step 4: Compute core as collection of those $c_{ij}$with single element.

Step 5: Delete those $c_{ij}=$ or $c_{ij}$with nonempty overlapwith the core.

Step 6: Define $f_D(U,\mathbf{R}) = \wedge\{\vee(c_{ij})\}$ with c_{ij}leftafter Step 5.

Step 7: Compute $g_D(U,\mathbf{R}) = (\wedge\mathbf{R}1) \vee(\wedge\mathbf{R}2) \vee \cdot \cdot \cdot \vee(\wedge\mathbf{R}l)$ by $f_D(U,\mathbf{R}) = \wedge\{\vee(c_{ij})|$

Step 8: Output all reductions $Red_D(\mathbf{R}) =\{\mathbf{R}1, . . . ,\mathbf{R}l\}$.

Preliminaries

In this segment we examine meaning of Rough set theory, fuzzy set theory, joined meaning of fuzzy rough set theory with two significant idea level of dependency calculation and discernibility calculation.

- **Rough Set**

The idea of Rough sets was presented by Z Pawlak in his fundamental paper of 1982 (Pawlak 1982). It is a conventional theory gotten from essential exploration on intelligent properties of data frameworks. Rough set theory has been a system of database mining or knowledge discovery in social databases. In its theoretical structure, it is another territory of vulnerability science firmly identified with fuzzy theory. We can utilize rough set way to deal with find underlying relationship inside uncertain and loud data.

Rough sets and fuzzy sets are reciprocal speculations of classical sets. The estimate spaces of rough set theory are sets with numerous enrollments, while fuzzy sets are worried about fractional participations. The quick advancement of these two methodologies gives a premise to "soft computing, " started by Lotfi A. Zadeh. Soft Computing incorporates alongside rough sets, at any rate fuzzy rationale, neural organizations, probabilistic thinking, conviction organizations, machine learning, transformative computing, and bedlam theory.

  - **Goals of Rough Set Theory**

The fundamental objective of the rough set analysis is the enlistment of (learning) approximations of ideas. Rough sets establish a sound reason for KDD. It offers numerical instruments to find patterns covered up in data.

It very well may be utilized for highlight choice, include extraction, data reduction, choice standard age, and pattern extraction (layouts, affiliation rules) and so forth

Distinguishes incomplete or all out conditions in data, takes out excess data, and offers way to deal with invalid values, missing data, dynamic data and others.

- **Information Framework**

In Rough Set, data model data is put away in a table. Each column (tuples) addresses a reality or an article. Frequently the realities are not predictable with one another. In Rough Set wording a data table is called an Information System..

Approximations

It is a formal approximation of a crisp set defined by its two approximations – ***Upper approximation*** and ***Lower approximation***.

- **Upper approximation** is the set of objects which possibly belong to the target set.

$\overline{R}X = \cup\{Y \in U/R : Y \cap X \neq \Phi\}$

- **Lower approximation** is the set of objects that positively belong to the target set.

$\underline{R}X = \cup\{Y \in U/R : Y \subseteq X\}$

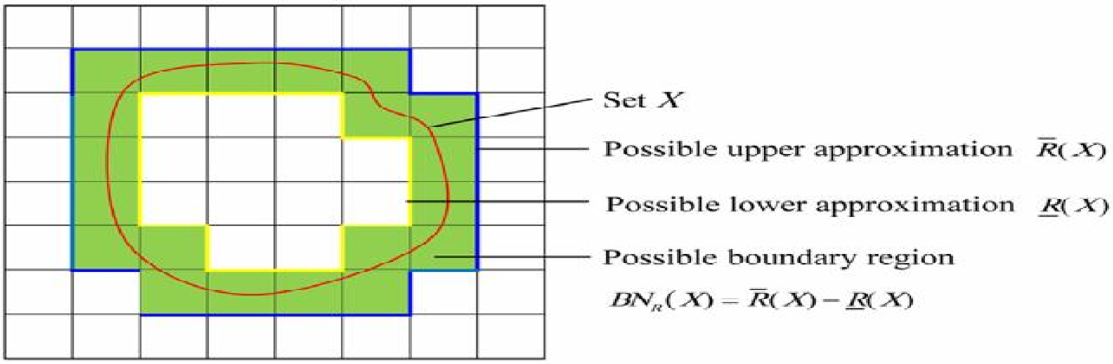

Figure 1: Rough set approximations

- **Positive region- Positive region can be expressed as**

Let (U, C ∪ D, V, f) be a decision system with Q ∈ D as a decision attribute. Its equivalence classes $[x]_{R_Q}$ are called decision classes. Given P ⊆C , then P is positive region (POS_P) comprises those objects from U for which the values of P allow to predict the decision class clearly [51]

$POS_P(Q) = U_{x \text{ x }} R_P \downarrow [x]_{R_Q}$

, if x ∈POS_P when object have it will also belong to the same decision class as x if the attributes in P have the same values as x. The following value (degree of dependence of Q on P) is used to determine the predictive potential of the attributes in P with respect to Q:

$\gamma_P(Q) = \frac{|POS_P(Q)\ \ |}{|U|}$

- **Fuzzy Set Theory:** Since rough set theory alone is insufficient to handle attributes relating to uncertainty with real or continuous values, a hybrid rough set and fuzzy set approach was created to handle information uncertainty concepts. LoftiZadeh pioneered the use of fuzziness in set theory to deal with concept ambiguity. In fuzzy set an element has a degree of belongingness to a set in particular, d (0 ≤ d ≤ 1). When X is a set and y is an element, the fuzzy membership function is represented as: $\mu_X(y) \in < 0, 1 >$.

Operations For two fuzzy sets A1, A2 in X

A1= {(x1, 0.1), (x2, 0.3), (x3, 0.6), (x4, 0.8)}

A2= {(x1, 0.5), (x2, 0.1), (x3, 0.7), (x4, 0.4)}

Then

Union of A1 and A2 =max {$\mu_{A1\cup A2}$ (x)**}** ={(x1, 0.5), (x2, 0.3), (x3, 0.7), (x4, 0.8)}

Intersection of A1 and A2 =min {$\mu_{A1\cap A2}$ (x)**}** ={(x1, 0.1), (x2, 0.1), (x3, 0.6), (x4, 0.4)}

Complement= μ_{A1} c (x) **= 1-** μ_{A1} (x) ={(x1, 0.5), (x2, 0.7), (x3, 0.3), (x4, 0.2)}

Cardinality of two fuzzy set is given by $|A1| = \sum_{x\in X}\mu_{A1}(x)$

- **Intuitionistic Fuzzy Set**

Let U ($\neq\varphi$) be an universe of discourse ofobjects. An intuitionistic fuzzy set B in U is collection of objects represented in the form $B =\{<x, l_B(x), m_B(x)> | x \in U\}$, where l_B: $U \rightarrow [0, 1]$and m_B: $U \rightarrow [0, 1]$ are called degree of membership and degree of non-membership of the element x respectively, satisfying $0 \leq l_B(x) + m_B(x) \leq 1$, $\forall x \in U$. $\pi_B(x) = 1 - l_B(x) - m_B(x)$ represents the degree of hesitancy of x to B. It is obvious that$0 \leq \pi_B(x) \leq 1$, $\forall x \in U$.

Any fuzzy set $B = \{<x, l_B(x)> | x \in U\}$ can be recognizedas a particular case of intuitionistic fuzzyset in the form $\{<x, lB(x), 1 - lB(x)> | x \in U\}$. Therefore an intuitionistic fuzzy set is considered as anextension of fuzzy set.The cardinality of an intuitionistic fuzzy set B isdefined by [56]:

$$|B| = \sum_{x\ U} \frac{(1 + l_B(x) - m_B(x))}{2}$$

Intuitionistic Fuzzy Rough feature Selection (IFRFS)

In 1998, Chakrabarty et al. [23] proposed a concept to design an intuitionistic fuzzy rough set (IFRS)(L, M) of a rough set (A, B), where L and $M$are both intuitionistic fuzzy sets in U (non-empty set of objects) such that $L \subseteq M$, *i.e.* $\mu_L \leq \mu_M$and$\upsilon_L \geq \upsilon_M$. In this case, lower approximation L and upper approximation M are both intuitionistic fuzzy sets. In 2001, Samanta and Mondal [46] proposed their method to define IFRS, where they defineda couple (E, F) as intuitionistic fuzzy rough set such that E and F are both fuzzy rough sets (as proposed by Nanda and Mujumdar [44]) and $E \subseteq$Complement (F). From [44], it is obvious that IFRSis a generalization of an intuitionistic fuzzy set, in which membership and non-membership functions are fuzzy rough sets. In 2002, Rizvi et al. [45] reportedtheir proposal as rough intuitionistic fuzzy set, which also contains hesitation margin on lower and upper approximations. In 2003, Cornelis et al. [24] defined the lower and upper approximations of $X \subseteq U$ (Universe of discourse)as follows:

$$R \downarrow_I X(y) = \inf_{x\ U} I(R(x, y), X(x))$$

$$R \uparrow_T X(y) = \sup_{x\ U} T(R(x, y), X(x)), \quad x, y \quad U.$$

where, T, I, R are an intuitionistic fuzzy triangular norm, an intuitionistic fuzzy implicate and an intuitionistic fuzzy relation on U respectively. Here, apair ($R \downarrow I\ X(y)$, $R \uparrow T\ X(y)$) represents intuitionisticfuzzy rough set.

However, all above proposed definitions do not consider memberships and non-memberships of individual objects to obtain the approximations. From literature [16, 24, 35], we can define intuitionistic fuzzy lower and upper approximations by considering individual objects as follows: A relation is said to be an intuitionistic fuzzy tolerance relation if it is reflexive and symmetric [54].Now, we define an intuitionistic fuzzy tolerance relations follows:

$$Let\ \alpha = 1 - \frac{|\mu_a(x) - \mu_a(y)|}{|\mu_{amax} - \mu_{amin}|},\ \beta = \frac{|\nu_a(x) - \nu_a(y)|}{|\nu_a max - \nu_a min|}$$

Then,

$$\{\mu_{Ra}(x, y), \nu_{Ra}(x, y)\} = \begin{cases} (\alpha, \beta)\ if\ \alpha + \beta \leq 1 \\ (1, 0)\ if\ \alpha + \beta > 1 \end{cases}$$

Where, $\mu_{Ra}(x, y)$ and $\nu_{Ra}(x, y)$ are membership and non-membership grades of intuitionistic fuzzy tolerance relation between x and y. μ_{amax},μ_{amin}andν_{amax}, ν_{amin} represent maximum and minimum membership and non-membership grades for attribute a. If R_P[24, 39] is the intuitionistic fuzzy tolerance relation induced by the subset of feature P, then,

$$<\mu_{Rp}(x, y), \upsilon_{Rp}(x, y)> \; = \inf_{a \in P} <\mu_{Ra}(x, y), \upsilon_{Ra}(x, y)>$$

Any object does not belong to the positive region, only if the equivalence class, it belongs to, is not an element of the positive region. Therefore, the degree of dependency can be defined by [63]:

$$\gamma''_P(Q)=\frac{\sum_{x\in U}\frac{1+\mu_{POS_{R_P}(Q)}(x)-\upsilon_{POS_{R_P}(Q)}(x)}{2}}{|U|}$$

where, $|U|$ = Cardinality of U. Now, we can calculate the reduct set by using concepts from [35].

- **Reduction Method**

Table 1: Example Dataset

x∈U	a	b	c	d	e
0	1	0	2	2	0
1	0	1	1	1	2
2	2	0	0	1	1
3	1	1	0	2	2
4	1	0	2	0	1
5	2	2	0	1	1
6	2	1	1	1	2
7	0	1	1	0	1

An example dataset[17] (Table I) will be used in this table there are four conditional attributes a, b, c , d, and one decision attribute e with 8 instances. Feature selection is reducing features over the conditional attributes to get reduced data set. Feature selection is the process of reducing the input variable to available model by using only relevant data and getting rid of noise in data. A reduct is defined as a subset of minimal cardinality $R_{\mathbf{min}}$ of the conditional attributes C such that $\gamma_{R_{min}}(D)=\gamma_C(D)$

$R = \{ X: X \subseteq C, \gamma_X(D)= \gamma_C(D)\}$

$R_{\mathbf{min}} = \{ X: X \in R, \forall Y \in R, |X|\leq|Y|\}$

Using the example, the dependencies for all possible subsets of C can be calculated

$\gamma_{\{a,b,c,d\}}(\{e\})=8/8$ $\quad\gamma_{\{b,c\}}(\{e\})= 3/8$

$\gamma_{\{a,b,c,d\}}(\{e\})=4/8$ $\quad\gamma_{\{b,d\}}(\{e\})= 8/8$

${}_{\{a,b,d\}}(\{e\})=8/8$ $\quad\gamma_{\{c,d\}}(\{e\})= 8/8$

${}_{\{a,,c,d\}}(\{e\})=8/8$ $\quad{}_{\{a\}}(\{e\})= 0/8$

${}_{\{b,c,d\}}(\{e\})=8/8$ $\quad{}_{\{b\}}(\{e\})= 1/8$

${}_{\{a,b\}}(\{e\})=4/8$ $\quad\gamma_{\{c\}}(\{e\})= 0/8$

${}_{\{a,c\}}(\{e\})=4/8$ $\quad{}_{\{d\}}(\{e\})= 2/8$

${}_{\{a,d\}}(\{e\})=3/8$

Note that the given dataset is consistent since ${}_{\{a,b,c,d\}}(\{e\})=1$

The minimal reduct set for this example is $R_{\mathbf{min}}=\{ \{b,d\}, \{c,d\}\}$

Discernibility Matrix-Based Fuzzy Rough Attribute Reduction

Attribute reduction based on discernibility matrix in IFDS.[18]

Input: IFDS= (U, C ∪ D, V, f)

In IFDS, all relative reducts are output.

Step 1: Calculate the maximal consistent block of every condition attribute;

Step 2: Calculate the similarity class of every object in the universe in terms of C;

Step 3: Calculate the generalized decision of every object in the universe

Step 4: In IFDS, create the discernibility matrix M;

Step 5: In IFDS, create the discernibility function f(M);

Step 6: Confirm

Some important application areas of feature selection

- Medical image processing
- Pattern recognition
- System monitoring
- Signal processing

- Text classifications
- Clustering
- Rule induction
- Bioinformatics

Limitations of Fuzzy Rough Set

Vagueness and ambiguity are two separate methods. A membership function is applied to the fuzzy set objects, which assigns a membership value between 0 and 1. This value specifies whether the object is likely to belong to a collection or is only occasionally connected to one. The disadvantage is that it takes a long time to compute; however, both have certain parallels, even though rough sets are computationally expensive.

Table 1:-Feature selection techniques based on fuzzy rough set theory

Authors	Purpose	Illustration
Sheeja, T. K., & Kuriakose[41]	Feature selection	This paper deal with very new approaches based on divergence measure of fuzzy rough set
Shi Qiang Wang et al[42]	Feature selection	This paper presents feature selection techniques for radiation source signals
R.B. Bhatt, M. Gopal[43]	Feature selection	Improve reliability of fuzzy rough sets algorithm
Richard Jensen and Qiang Shen[39]	Feature selection	Similarity relation based fuzzy rough sets approach proposes with discernibility matrix
A. Chouchoulas and Q. Shen[44]	Keyword reduction for text categorization	Fuzzy rough set based approaches that deal with different techniques of information classification and information retrieval.
Martine De Cock, Chris Cornelis, and Etienne E. Kerre[45]	The Forgotten Step	Soft similarity classes
A.K. Tiwari et al[35]	Feature selection	This paper present new concept Intuitionistic fuzzy rough sets that deal with pair of membership values and non membership values
Didier DUBOIS Henri PRADE[4]	Vagueness for fuzzy sets and coarseness for rough sets	Laying Rough Sets And Fuzzy Sets Organized
ZHU AND YANG[46]	A mobile robot can adequately sense the environment around	Neural network techniques with neurofuzzy based system for mobile robot navigation in unknown environments
Ivo Düntsch et al[47]	Feature selection	Rough set data analysis
Lei Yu and Huan Liu[48]	Decouples relevance analysis and redundancy analysis	Focuses on new definition of redundancy analysis and relevance analysis
Richard Jensen and Qiang Shen[49]	Dimensionality Reduction	Semantics-Preserving Dimensionality Reduction
Y.Saeys et al.[50]	Feature selection	A review paper on feature selection techniques ,in the field of bioinformatics
Richard Jensen and Qiang Shen[2]	Attribute selection	FRFS
Qinrong Feng and Rui Li[33]	Attribute selection	Based on similarity relation Intuitionistic Fuzzy Decision Systems develop and discernibility matrix based approach follow to find reduct
PAT LANGLEY[51]	Feature selection	Feature selection in Machine Learning
A.K. Tiwari et al.[36]	Feature selection	A new model intuitionistic fuzzy-rough set developed with its application
PawanLingras and Richard Jensen[38]	Supervised learning, feature selection, and unsupervised learning.	Rough and Fuzzy Hybridization
ZdzisławPawlak.[52]	Specific additions of the rough set methodology	Rough sets
A.M. Radzikowska, E.E. Kerre[53]	Fuzzification of rough sets	Fuzzy Sets and Systems
A. Lasisi et al[54]	Fuzzy rough feature selection	artificial immune recognition system with mining agricultural data used for feature selection
Q. Shen, A. Chouchoulas[55]	Feature selection	A rough-fuzzy approach for generating classification rules

Andrzej SKOWRON et al[56]	Reducts, core and dependencies generation	The Discernibility Matrices And Functions In Information Systems
J. Błaszczyn´ski et al[57]	Rule induction	Consecutive casing rule induction algorithm for variable steadiness rough set approaches
R.W. Swiniarski, A. Skowron[58]	Feature selection in pattern recognition	Rough set procedures in feature selection and appreciation
Tsang, G. C., Degang, C et al[59]	Discernibility matrix based approach is used calculate degree of dependency to find reduct sets	On attributes reduction with fuzzy rough sets
WANG Jue, WANG Ju[60]	Ordered Attributes Method	Reduction Algorithms to find reduct sets Based on Discernibility Matrix
Richard Jensen and Qiang Shen[39]	Feature selection	Novel Methodologies to Fuzzy-Rough Feature Selection
W.-Z. Wu et al[61]	(I, T)-fuzzy rough approximation operators	On categorisations of (I, T)-fuzzy rough guesstimate operators
XiuyiJiaa , Lin Shang et al[20]	Attribute reduct with user prefrence	Generalized attribute reduction in rough set theory
Daniel S. Yeung et al[19] L. A. Zadeh[62]	Approximation operators Approximation	On The Simplification Of Fuzzy Rough Sets The Impression Of A Linguistic Variable And Its Application To Ballpark Reasoning
Jensen and Shen, in [2004]	attribute reduction technique	To Deal With Problem Of Web Categorization
Neil Mac Parthalain et al[63]	simultaneous instance and feature selection	Eliminates instances and features from a dataset simultaneously, that appear fuzzy clauses generated from the data
J. Dai et al[64]	incomplete interval-valued information systems	a dominance-based fuzzy rough set approach
A. Lasisi et al.[54]	Mining agricultural data	Coupled CLONALG and AIRS
Abhishek Jhawar et al[65]	instance selection	Detecting erroneous gait patterns or deviations
ArunkumarChinnaswamy et al[66]	Dimensionality reduction	A rank based information gain filter is used
Richard Jensen and Qiang Shen[67]	extension of the fuzzy-rough feature selection	interval-valued fuzzy sets
C. Cornelis et al[68]	attribute selection	multi-adjoint fuzzy rough sets
Yiyu Yao[69]	procedures altered annexation relations and operations on interim sets	granular computing concentrations on a multilevel and multi-view granular structure.
Q. Hu et al[70]	soft fuzzy rough sets	reduce the influence of noise
R. Jensen, N. Mac Parthaláin[71]	Reduce computational effort in attribute selection	vicinity guesstimate step and attribute alliance

Conclusion

This review paper systematically studies attributes reduction based on intuitionistic fuzzy rough sets. We introduce some concepts and theorems of attributes reduction with intuitionistic fuzzy rough sets, and completely investigate the structure of attributes reduction. By employing the approach of discernibility matrix, an algorithm to find all the attributes reductions is also presented. This review paper has introduced a fuzzy–rough technique for quality reduction which lightens significant issues experienced by conventional RSAR like managing clamor and genuine esteemed attributes. This epic methodology has been applied to help classification of web content, with promising outcomes. Specifically, while holding less attributes than the regular fresh rough set-based strategy, the work involves the classi6ers that utilize the held attributes to have a higher classification rate.

Considering the meaning of characteristic reduction in the rough sets model, In this review paper principally centers around quality reduction dependent on IF rough sets. Subsequent to reviewing trait reduction with conventional rough sets, a few ideas and hypotheses of property reduction dependent on IF rough sets are presented and the design of reduction is totally contemplated. An algorithm dependent on the discernibility grid to register all the trait reductions is created, and the ideas of quality reduction are exhibited by a model.

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