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RESPONSE OF THERMO-ELASTIC MEDIA UNDER DYNAMIC LOADINGS ON A CIRCULAR CAVITY

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ABSTRACT

The elastic deformation due to time dependent surface traction (from the Duhamel-Neumann Analysis) can be determined without reference to its Thermal State. This procedure is adopted in the solution of the dynamical problem in classical theory of elasticity. The elastic constants which Appear in the equations of motion are defined under adiabatic conditions and since elastic waves being non dispersive, produce no increase in the entropy of the solid. No inconsistency seems to arise. This discription of elastic wave propagation is physically over simplified. In fact a change in volume must cause the temperature as well as the stresses to change. Coupled thermo-elastic wave problem concerning the region outside a circular cavity in a homogeneous and isotropic elastic media under mechanical blast loadings as special case has been considered. The expressions for stresses and temperature are obtained.

Keywords: *Elasticity, Thermo-Elastic, Isotropic Elastic, Oscilation.*

Introduction

When a longitudinal wave passes through a solid, the elements are successively compressed and dilated. These phenomena are accompanied respectively by heating and cooling. Since the thermal conductivity of the solid is non-zero and the disturbance has a finite frequency, the source of energy will be converted into heat energy during the first half of an oscilation and will not be recovered during the dilation phase.

With the foregoing remarks in mind this paper is prepared. A formal solution of coupled thermo elastic problem of transient wave diffraction in an infinite elasic media having one circular cavity under blast loadings of intensity P_0 (as special case) has been discussed.

Coupling was postulated by J. DUHAMEL and generalized conduction equation which we use was derived by W.VOIGT and JEFFREYS. A few problems are solved on Thermo-Mechanical coupling effects with cavities KUMAR (1987) and uncoupled problem ERINGEN (1961). The result of ERINGEN (1961) are obtained as particular case.

Thermal Coupling

In the development of the general theory of propagation of thermo elastic waves varying harmonically with the time a number of particular problem have been solved. There have bee mainly typical problems of classical elasto-kinetics generalized and extended within the frame work of thermo elasticity, considerable attention has been devoted to surface waves. This problem was first solved by LOCKETT [1958, 1959] and then more thoroughly by CHARDWICK AND WINDLE [1964].

In devising a theory of surface waves in a plane state of deformation, we start from the wave equation for longitudinal and transverse waves and the equation of thermal conduction. The wave travels parallel to the plane bounding the semi space and decrease in amplitudes with the distance from this plane. It is assumed that either the stress and temperature or the stress and heat flux are zero on this plane. An algebric equation of the third order with coupled co-efficients is obtained from the determinant of the system of equation expressing the homogeneous boundary conditions and one of the roots of this equation gives the plane velocity by a surface waves. Certain inequalities satisfied by this solution show

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that the surface wave undergoes damping and dispersion and that its velocity is less than that of the longitudinal or transverse waves. NOWACKI and SOKOLOWSKI [1959] have investigated in a smiliar manner this propagation of a harmonic waves in a thermoelastic layer. They consider both symmetrical and anti-symmetrical (Flexural) form of wave for two thermal condition on the boundary T = 0 and T, n = 0; owing to the small value of the parameter characterising the thermo-elastic medium an approximate solution of the transcendental equation was obtain by perturbation method.

The propagation of harmonic waves in a infinite circular and in a thick walled pipe has been studied by LOCKETT [1959]. J. INGACZAK and NOWACKI [1961] have examined the forced vibration of an infinite cylinder with rectangular cross-section exited by heating the cylindrical surface. The same authors have solved in [1961] the problem of forced longitudinal vibration in dises and flexural vibrations produced by heating of plates CHARIDWICK (1962) has investigated the same problem. J.INGAIZAK and NOWACKE [1962] have discussed the problem of the propagation of thermo-elastic plane wave in an infinite medium with a spherical or cylindrical cavities. The plane wave which has been produced by a plane heat source flows around the cavity, the temperature field undergoes a disturbance and a concentration of temperature and stress take place in the neighbourhood of the cavity. A partial solution is obtained in a closed form and the residual solution is expressed as an infinite series with complex co-efficients.

From the results obtained by Numerical calculation on the difference between the uncoupled and coupled solution for a solid sphere is recognised to be larger than that for an infinite medium with spherical cavity. Particularly with regard to a solid sphere the following fact is obtained, even for usual industrial materials, i.e., alluminium alloy the difference between the values of the uncoupled and coupled stresses amounts to about 10 percent is some portions of the sphere at certain time. Maximum values of the coupled thermal stresses become larger coupling co-efficient and a smaller poisson's ratio. Moreover, under consideration of thermo elastic coupling effect, it can be explained that an abiadatic change of volume and a variation of temperature occur even in the region where heat flow the boundary has not reached.

Thermo-Mechanical Coupling

According to the available references on thermomechanical coupling, LESSOON [1957] discussed the problem of sudden loading applied to the boundary of a spherical cavity in an infinite medium applying the perturbation method and CHARDWICK [1960] applying the asymptotic method for small times. J. INGACZAK and NOWACKI [1961] have solve the problem of forced longitudinal vibration in discs and flexural vibration produced by loading of plates. Several solution have been obtained corresponding to the so-called 'Lamb Problem' of classical elasto-kinetics. These problems concerns the influence of loading on a semi-infinite thermo elastic medium. Two typical problems of this type was solved by NOWACKI [1960], first the case of axially symmetrical loading, second the case when loading produce a plane state of deformation. NOWACKI [1962] has solved the problem of periodic concentrated force acting in the direction of tha Z-axis at the origin of the cylindrical co-ordinate system. The action of this force results in propagation of both longitudinal and transverse waves. The problem is axisymmetric with respect to Z-axis. The solution are expressed in the form of Hankel integrals.

Recently the fourth International Congress on Thermal stress 1s joining to be held in June 2001 at Osaka Profecture University in co-operation with Osaka Institute of technology Osaka, Japan and a good number of references on thermal stresses will be available in the proceedings of the fourth International Congress TS 2001.

Response of Thermo-Elastic Media under Dynam Loadings on a Circular Cavity

- **Mathematical Formulation and Solution of the Problem**

The origin of the plane polar co-ordinates (r,q) is taken at the centre of the cavity. The boundry of the cavity r = a is taken at zero temperature and is under dynamical tractions.

Subject to further conditions that the stress waves generated at the cavity are advancing into the medium r > a (i.e. divergent waves) and vanish at

The coupled equation of heat conduction and motion in the absence of body forces and of internal heat generation are expressed as

.....(2.1.2.1)

.....(2.1.2.2.)

where is displacement vectar, and are Lame's constant; k is Thermal diffusivity, is density

To being absolute temperature of the medium in stress-free state and is the thermal capacity at constant volume, is linear thermal expansion, is time,

The boundary condition of the problem are

.....(2.1.2.3)

Where and are arbitrary function of and *t*. T is temperature.

The stress strain relations are

.....(2.1.2.4)

where is the kronecker delta; is strain components.

Now introducing the following dimensionless quantities.

The above equation (2.1.2.1) and (2.1.2.2) thus reduces to

.....(2.1.2.5)

and .....(2.1.2.6)

where

is a coupling factor.

The displacement vector can be taken as

.....(2.1.2.7)

Where and are scalar and vector potential function respectively.

Substituting (2.1.2.7) into (2.1.2.5) & (2.1.2.6), we have

.....(2.1.2.8)

.....(2.1.2.9)

.....(2.1.2.10)

Where

Eliminating or from (2.1.2.8) and (2.1.2.9)

.....(2.1.2.11)

We define a transformation SNEDDON (1951)

(a)

and inverse transformation

(b)

..... (2.1 2.12)

where 'Bar' Stands for transformed function and for transform variable.

Operating the transform (2.1.2.12 (a)) on (2.1.2.11), we get

.....(2.1.2.13)

or it can be written as

.....(2.1.2.14)

where , are the roots of the equation.

(a)

(b)

.....(2.1.2.15)

Long Time Solution

If we expand the values of , and , of the above expression (2.1.2.15b) for small value of in Taylor's series we get a long time solution.

(2. 1. 2. 16)

Neglecting the higher powers of , we have

where

(2. 1. 2. 17)

After taking the transform of equation (2. 1. 2. 10) and substituting the values of and from (2. 1. 2. 17) in (2. 1. 2. 14).

We get

(a)

(b)

(c)

(2. 1. 2. 18)

Writing the transformed potentials as

(a)

(b)

(c)

(2. 1. 2. 19)

and the solution of the above equation (2. 1. 2. 18 a ,b, c) for divergent waves are|

(a)

(b)

(c)

(2. 1. 2. 20)

where are arbitrary constants :

and are Hankel function of first kind and modified Bessel function of second kind re- spectively of order n.

As the transformed thermo-elastic potential functions are determined the corresponding components of stresses and temperature can be calculated. They are

(2. 1. 2. 21)

(2. 1. 2. 22)

(2. 1. 2. 23)

can now be evaluated with the help of the boundary conditions

(2. 1. 2. 3)

- **Dynamic Tractions Applied to Cavity**

The transformed tractions and are expanded into fourier series and equating them to and . Again we take and as finite fourier transform of cavity traction such as

.....(2.1.3.1)

Applying boundary Condition (2.1.2.3) after taking transform and form (2.1.3.1). The constant may be computed. Thus the transformed potential functions become.

.....(2.1.3.2)

.....(2.1.3.3)

.....(2.1.3.4)

when

.....(2.1.3.5)

The transformed stresses and temperature are obtained by substituting (2.1.3.2), (2.1.3 3) and (2.1.3.4) in their respective equations.

(2. 1. 3. 6)

.....(2.1.3.7)

.....(2. 1. 3. 8)

.....(2.1.3.9)

The above stress components , and temperature T may be evaluated from the inversion integral (2.1.2.12b) i.e.

Similarly all other components and temperature can be determined. This formally completes the problem we have give some special cases which are of practical interest.

- **Special Cases**

Blast Loadings and Surface Shear is Zero

.....(2.1.4.1)

and thus taking their transform

; .....(2.1.4.2)

Then

.....(2.1.4.3)

for all .....(2.1.4.4)

The expressions for temperature field and displacements are

.....(2.1.4.5)

.....(2.1.4.6)

.....(2.1.4.7)

where

where dash stands for differentiation.

.....(2.1.4.8)

Discussion of Results

We have discussed the problem under thermo-mechanical coupling effects and due to this coupling, we have got one more scalar potential function and compared to the uncoupled problem. The function is responsible for thermal waves into the medium which developed due to the conversion of some parts of Mechanical-energy into thermal energy. Due to this thermal waves. We get some extra terms tn all expressions compared to uncoupled elastodynamic problems.

As a matter of fact consideration of 'Thermo-Mechanical coupling effects is desirable for all elasto-dynamic problem at least for long time solutions or high velocity or force impact problems otherwise we neglect a good amount of energy supplied to the medium.

The coupling of strain and temperature fields is done by coupling factor and if the temperature field developed in the medium vanishes from equation (2.1.3.6) and displacements and stresses from equation (2.1.3.7), (2.1.3.8) etc. agree with the equation (15) or ERINGEN (1961).

For special case if the temperature from (2.1.4.5) vanishes and displacements agree with corresponding term of ERINGEN (1961).

if , , the above problem reduces to stationary, uncoupled problem.

Nomenclature

= Any Constant

= Cartesian Co-ordinates

= Polar Co-ordinates

= Lame's constants

= Co-efficient of linear thermal expansion

= Scalar and vector potential function

K = Thermal diffusivity

= Density of the medium

= Young's modulus

G = Shear modulus

C_V = Thermal capacity at constant volume.

= Poisson's ratio

= Stress Components

e = Dilation

= Strain Components

= Kronecker delta

= Displacement vector

= Absolute temperature

T = Temperature

P = Intensity of loading

= Transform variables

u (t) = Unit function
F(t) = Arbitrary function of time
= Continuous function
= Dirac Delta function
= Hankel function of ordern
= Spherical Hankel function of order *n*
= Legendre polynomial of order *n*
= Bessel function of order *n*
= Modified Bessel function of order *n*
= Dilational velocity of elastic waves
= Rotational velocity of the elastic wave
= Coupling factor
= Stress intensity of the incoming shock waves
H(t) = Unit step function
S = Specific entrophy
u = Specific internal energy
= Resultant body force
= Young's modulus
= Poission's ratio
= Hreat fulx vector
= Finite fourier transform of cavity traction

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