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A SURVEY STUDY OF ASSOCIATION BETWEEN GRAPH AND GRAPH LABELING TECHNIQUES

V. T. Dhokrat*
P.G. Jadhav**
S.B. Gaikwad***

ABSTRACT

Graph theory is a well-known prosperous branch of Mathematics. Mathematics gives the same name to different things. Graph theory creates graph labeling techniques in which different labeling is given to the same graphical structures. The graph is a mathematical structure describing dots, curves, and bars. Graph labeling is the venture of integers to the vertices or edges, or each, subject to certain conditions. The concept of graph theory is used to shape various mathematical models for applications in Operation Research, control, and Engineering; especially in studies areas of laptop technological know-how-which include statistics mining, photograph segmentation, clustering, photograph capturing, and networking that is used in structural fashions. Depending on the trouble scenario, a type of graph is used to represent the hassle, and by applying an appropriate graph labeling technique the hassle may be solved. Graph labeling is a flourishing as well as application-oriented area of research in the field of graph theory. In this paper, we study the different graph labeling techniques. In mathematics, we have different relationships between numerals. We use these relationships for labeling the vertices and edges in a graph. Accordingly, there are various graph labeling techniques that we consider in this paper viz. Graceful, Cordial, Magic, Mean, Sumlet labeling, and so on. For each labeling technique, we use different relationships. Again these labeling technique contains sub labeling techniques depending on some variation in a mathematical relation which is used earlier. To date different papers are available, so I collect some of these and study them. This article helps researchers how we can label graphs using different labeling techniques, what are the different types of labeling, and various examples of each labeling technique, one graph can be labeled by using different labeling techniques. In this paper, I consider different graphs for different labeling so that the researcher who is not familiar with these can at least become familiar with them.

Keywords: *Graph, Graph Labeling, Graceful, Cordial, Magic, Mean, Radio, Power Mean, Permutation, Combination, Graph, Sumlet Labeling.*
2010 Mathematics Subject Classification: Primary 05C38, 05C76. 05C78, 05C22

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Introduction

The Konisberg Seven Bridge problem which is like a puzzle has no relationship with mathematics is solved by a great mathematician Euler in 1735.The solution to this problem led to the formation of a new branch of mathematics known as graph theory. A graph is a geometrical structure which consists of "vertices" and lines segments joined by these vertices known as edges. Graph labeling

* Department of Mathematics, K.T.H.M. College, Nashik, Maharashtra, India.
** Department of Mathematics, Dnyaneshwar Gramonnati Mandal Hon. Balasaheb Jadhav Arts, Commerce and Science College Addr: Ale, Ta: Junnar Dist: Pune, Maharashtra, India.
*** Department of Mathematics, New Art's, Commerce and Science College, Ahmednagar, Maharashtra, India.

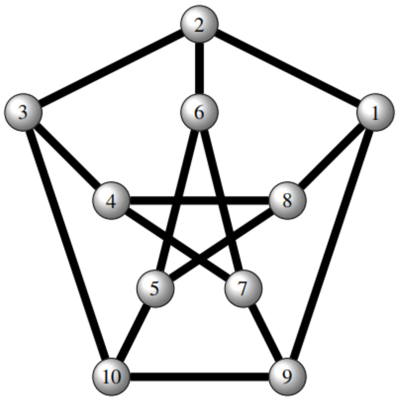
have been introduced by Alexander Rosa in the mid-1960s. Over the past seven decades, the area of graph labeling developed very fast. Graph labeling is a flourishing as well as application-oriented area of research in the field of graph theory. To date, more than 200 types of graph labeling had been studied. In this survey paper, we try to collect some important graph labeling techniques with suitable graphs. We consider only simple, undirected, finite and connected, graphs. We follow the terminology and notations of a graph from Harary's 'Graph Theory' book and for Graph labeling techniques follows Gallian's dynamic survey of graph labeling.

Graph

Definition: 1. A graph is an ordered triplet G = (V,E,F), consisting of a non-empty set V, whose elements are called vertices, a set E whose elements are called edges and an incidence function F that associates with each edge of G an unordered pair of vertices of G.

If e is an edge and 'r' and 's' are vertices such that F(e) = rs then 'e' is said to join r and s. The vertices r and s are called the end vertices of an edge e.

Exa. Peterson graph.

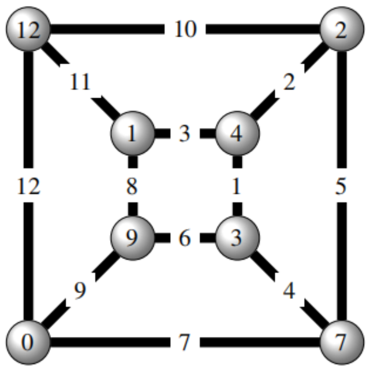


Peterson graph

Graph Labeling

Definition: 2. In a Graph, an assignment of non-negative integers to the vertices or positive integers to the edges, or both, subject to some particular conditions is known as Graph labeling.

Exa.

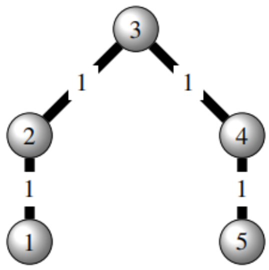


Examples of Graph having Graceful Labeling

- All paths.
- All symmetrical trees.
- Complete graph Kn for n less equals to 4.
- Wheel graph Wn for n ≥ 3.

Un-graceful Graphs: A simple graph G is said to be un-graceful if there does not exist any graceful labeling.

Exa.



Types of Graceful Labeling of a Graph

- Edge-Graceful.
- Vertex-Graceful.
- Arc-graceful labeling and k-arc-graceful labeling of Digraphs.
- Fibonacci graceful graph.
- Even graceful.
- Odd graceful.

Result

Graceful graph with m (m is positive integer) number of edges and m+1 number of vertices has the following characteristics:

- A single graph possesses a distinct triangle graph.
- A graph having m edges, the vertices labeled with 0 and m are always adjacent.
- Triangular graph may present in the graceful graph.
- In a graceful graph if we interchange every vertex label q with q−k, the resulting labeling is also graceful. The end vertices of an edge with labels a and b become m−a and m−b .
- Subgraph of a graceful graph need not be graceful.
- Super graph of a graceful graph need not be graceful.
- There may be at most m! graceful graph with m edges.

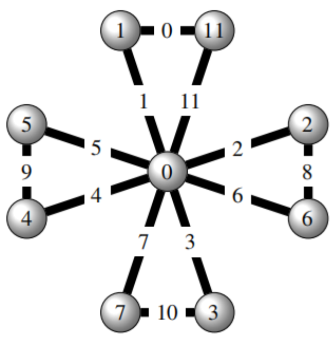
  - **Harmonious Labeling**

Graham and Sloane [33]

Definition: 4. Let G = G (V,E) be a connected graph with n vertices m edges. A function F is called as harmonious labeling of graph G if F : V → {0,1,2,...,m − 1}, is one-one and the induced function F′ : V → {0,1,2,...,m}, defined by F′(e = uv) = (F(u) +F(v))(modm) is both one-one and onto.

A graph that possesses harmonious labeling is called a harmonious graph.

Exa. F_4



Examples of a graph having Harmonious Labeling

- Peterson graph.
- Wheel graph Wn = Cn +K1 is harmonious.
- Kn (2) is harmonious if n = 4 but not harmonious if n is odd or n = 6.
- The Cycle Cn(n ≥ 3) is harmonious if and only if n is odd.
- All ladders except L_2, Friendship graph Fn is harmonious except n ≡ 2 mod4, the fan graph fn are harmonious.

Types of Harmonious Labeling of a Graph

- Odd harmonious Labeling.
- Even harmonious labeling.

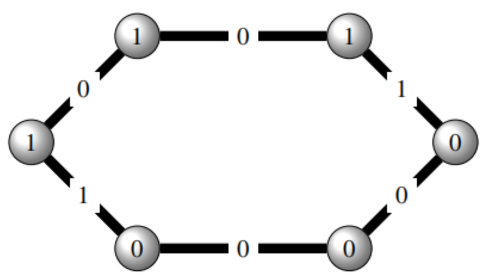
Result

- We can represent Harmonious labeling in different ways.
- If f is a Harmonious labeling of any graph G with m edges, than a f(x)+b is also harmonious labeling of G. where a is invertible element of set m and b is any arbitrary element of m. (Zm).
- Any vertex in a harmonious graph can be assigned the label 0.
- Trees with exactly two vertices are assigned the same vertex label.
- Every tree is harmonious.
- Complete graph is harmonious if and only if n ≤ 4.
- Let T be a harmonious labeled tree containing an edge e = uv labeled as f(u) + f(v) where v is a pendant vertex and f(v) is the repeated vertex label. If w is any other vertex in G, we may delete edge uv and vertex v and replace them with a new vertex z and edge e′ = wz where z is labeled with f (z) = f(u) + f(v)− f(y).
  - **Cordial Labeling**

  Cahit [9]

Definition: 5. Let G = G (V, E) be a connected graph with n vertices m edges. A graph is called cordial if it is possible to label its vertices with 0′ s and 1′ s such that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and similarly the number of edges labeled 0 and the number of edges labeled 1 differ at most by 1.

Exa. Cordial Labeling Graph of Shell Graph



Shell graph

Examples of a Standard Graph having Cordial Labeling

- Every tree is cordial.
- The complete graph Kn is cordial if and only if n ≤ 3
- The complete bipartite graph Km,n is cordial for all m and n
- The friendship graph $C_3^{(t)}$ (i.e. the one-point union of t 3-cycles) is cordial if and only if $t \not\equiv$ 2(mod 4).
- All fan graph fn are coordial.

- The wheel Wn is cordial if and only if n ≢ 3 (mod 4).
- Maximal outer planar graphs are cordial.
- An Eulerian graph is not cordial if its size is congruent to 2 (mod 4).
- Kuo, Chang, and Kwong determine all the values of m and n for which mKn is cordial.
- Every Skolem-graceful graph is cordial.
- A 3-regular graph of order n is cordial if and only if n ≢ 4 (mod 8).

Types of Cordial Labeling

- Difference cordial labeling.
- Edge product cordial labeling.
- Prime cordial labeling graph.
- Planar grid cordial labeling.
- Context of Duplication cordial.
- Second order cordial labeling.
- SET cordial labeling.
- Integer cordial labeling.
- Signed product cordial labeling.
- Mean cordial labeling.
- Geometric Mean cordial labeling.
- Harmonic Mean cordial labeling.

Result

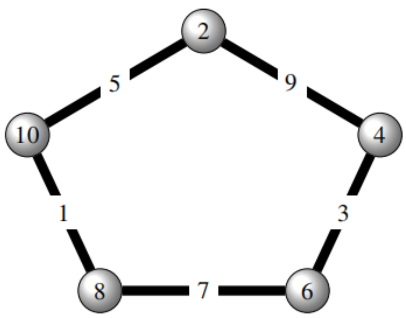
- Jewel graph Jn is cordial.
- DS (K1, n) is cordial graph.
- Jelly fish Jn,n is cordial. .
- Double fan DFn is a prime cordial graph for n = 8and ≥ 10.
- DS (Pn) is a prime cordial graph.
  - **Magic Labeling of a Graph**

    Magic labeling had been introduced by Sedlacek in 1963 [41] using the concept of magic squares in number theory,

- **Vertex magic Labeling of a Graph**

Definition: 6. Let G = G(V,E) be a connected graph n vertices m edges.A graph G is said to be vertex magic if there exists a bijective function F : V $\cup$ E → {1,2,...,m+n}, such that for all vertices of G, the sum of label on a vertex and the labels of its incident edges is constant such a bijection is called vertex magic labeling of G. A graph that admits vertex magic labeling is called a vertex magic graph.

Exa. Vertex magic labeling of cycle graph C5 with magic constant K=16

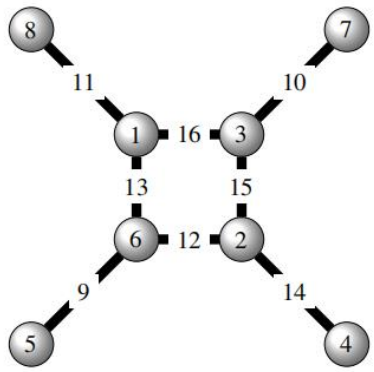


$5-cycle$

- **Edge Magic Labeling of a Graph**

Definition: 7. Let G = G(V,E) be a connected graph with n vertices and m edges.A graph G is said to be edge magic if there exists a one-one and onto function F : V $\cup$ E → {1,2,...,m+n}, such that for all edges uv of G, F(u)+F(v)+F(uv) is constant such a one-one and onto function is called edge magic labeling of G. A graph having edge magic labeling is called an edge magic graph.

Exa. Edge magic labeling of cycle graph C6 with magic constant K=20



Examples of a Graph having Magic Labeling

- Complete graph Kn is magic for n = 2 and n ≥ 5
- Complete bipartite graph Kn,n is magic for all n ≥ 3
- The friendship graph C3 (t) (i.e. the one-point union of t 3-cycles) is cordial if and only if t ≢ 2 (mod 4).
- The fan graph fn is magic if and only if n=odd number and n ≥ 3.
- The wheel graph Wn are magic for n ≥ 4 and Wn with one spoke deleted is magic for n=4 and n ≥ 6.
- A connected graph with m vertices and n edges other than P2 exits and is magic if and only if 5m/4 < n ≤ m(m−1)/2.

Types of Magic Labeling

- Semi-Magic graph.
- Super-Magic graph.
- Anti-magic graph.
- Prime-magic graph.
- H-magic graph.
- Magic labeling of type (a,b,c)
- Sigma labeling/ Distance-magic labeling

Result

- The star $K_{1,n}$ is strong magic for n = 1, ideal magic for n = 2,3,and weak magic for n > 3
- The graph G and H are magic if and only if GXH is a magic graph.
- The path Pn with n vertices has an edge-magic total labeling with magic constant (5n + 2)/2 when n is even and (5n+1)/2 when n is odd.
- Let G be a countable graph that has an infinite set of mutually disjoint edges and A is isomorphic to a countable subgroup of the real numbers under addition then for any k in A there is an edge-magic labeling of G with elements from A that has magic constant k.

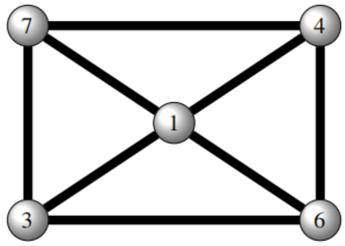
  - **Radio Labeling**

  Chartrand, Erwin, Zhang, and Harary [15].

Definition: 8. Let G = G (V,E) be a connected graph with n vertices and m edges. A radio labeling of a connected graph G is an injective function c from the vertex set V to the set of natural numbers such that d (u, v)+|c(u)−c(v)| ≥ 1+diam(G) for every two distinct vertices u and v of G.

The maximum number assigned to any vertex of G is known as radio number of c and is denoted by, r n(c), The radio number of G, r n(G), is the minimum value of r n(c) taken over all radio labelings c of G.

Exa. Radio mean Labeling of a Graph



w_4

Examples of Radio Labeling Graph

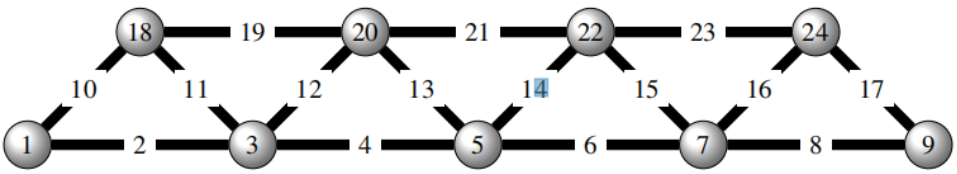
- The r n(G) of the sunflower graph SFn is its order.
- The r n(G) of a gear graph Gn is 2n+1.

  - **Mean Labeling of Graphs**

Somasundaram and Ponraj [40].

Definition: 9. Let G = G(V,E) be a connected graph with n vertices and m edges. A graph G is said to be a mean graph if there is an injective mapping F from the vertices of G to {0,1,2,−−−−−,m} such that when each edge (u, v) is labeled with (F(u) +F(v))/2 if F(u) +F(v) is even and (F(u) +F(v) +1)/2 if F(u) +F(v) is odd, All edge labels are distinct.

Exa.



Examples of Mean Labeling Graph

- Path Pn Cycle graph Cn, bipartite graph K2, n, triangular snakes, quadrilateral snakes etc. are mean graphs.
- Kn if n < 3
- K1, n if n < 3
- The friendship graph $C_3^{(t)}$ if t < 2

Types of Mean Labeling Graph

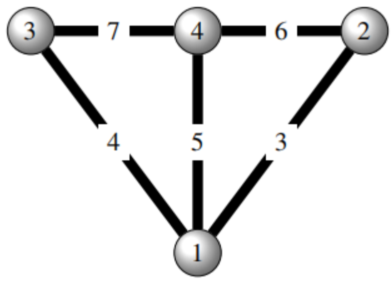
- Vertex even mean labeling.
- Vertex odd mean labeling.
- Super mean labeling graph.

  - **Irregular Labeling of Graph**

Chartrand, Jacobson, Lehel, Oellermann, Ruiz, and Saba [14].

Definition: 10. Let G = G (V,E) be a connected graph with n vertices and m edges. An irregular labeling of a graph G with no isolated vertices as an allotment of positive integer weights to the edges of G in such a way that the sums of the weights of the edges incident at each vertex are distinct. The minimum of the largest weight of an edge over all irregular labeling of a particular graph is called the irregularity strength of G and is denoted by s(G). If no such weight exists for an irregular labeling then s(G) = ∞.

Exa.

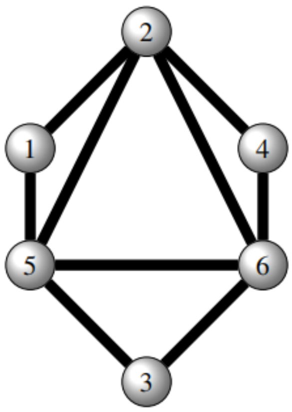


Prime Labeling of Graph

Tout, Dabboucy, and Howalla[20].

Definition: 11. Let G = G(V,E) be a connected graph with n vertices and m edges. A graph G is said to possess a prime labeling if there exists a bijection F : V → {1,2,...,m} such that for each edge(u, v), gcd (F(u),F(v)) = 1 i.e. (F(u) and F(v) are relatively prime.

Exa.



Examples of Prime Labeling of Graph

- Caterpillars with maximum degree at most 5.
- Paths, stars, complete binary trees, spiders etc.
  - **Combination and Permutation Labeling of Graph**

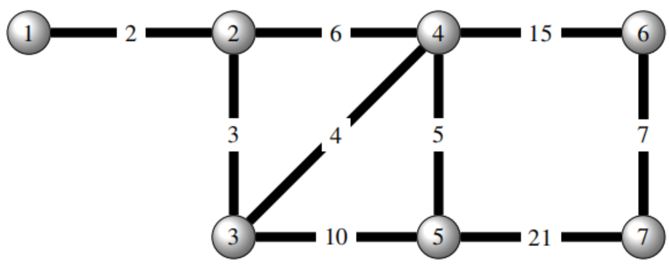
Hegde and Shetty [20]

Combination Labeling of Graph

Definition: 12. Let G = G (V,E) be a connected graph with n vertices and m edges. A function F: V → {1,2,...,m} is called as combination labeling of graph G if each edge (r,s.) is assigned the label (F(r))!/[F(r)−F(s)]!(F(s))! Where F(r) > F(s).

Here all edge labels are distinct.

Exa.



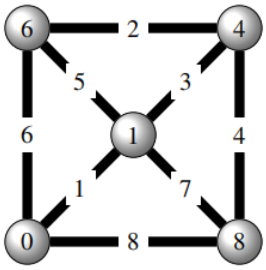
Examples of Combination Labeling of a Graph

- Kn for n ≤ 5
- Cn, n > 3.
- Kn, n if n ≤ 2.

Permutation Labeling of Graph

Definition: 13. Let G = G (V, E) be a connected graph with n vertices and m edges. A function F: V → {1, 2,..., m} is called as combination labeling of graph G if each edge (r,s) is assigned the label (F(r))!/[F(r)−F(s)]! Where F(r) > F(s).

Here all edge labels are distinct.



Examples of Permutation Labeling of a Graph

- Kn for n ≤ 5

Result

- If G is a permutation graph then G−e i.e. the graph obtained from G by deleting the edge e) is also a permutation graph.
- The graph k-wheel admits a permutation labeling for every integer n ≥ 2.
- The graph k-fan Fn,k.
- The gear graph G_{2n}.

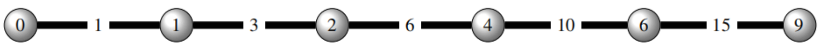
▪ **Triangular Sum Labeling of Graph**

Hegade and Shankaran [36]

Definition: 14. Let G = G (V,E) be a connected graph with n vertices and m edges. A labeling of the graph with m edges is called a triangular sum labeling if the vertices can be assigned with distinct non-negative integers in such a way that, when an edge whose vertices are labeled with i and j is labeled with the value i+ j.

All edge labels are different.

Exa.



P_6

Examples of Triangular Sum Labeling of Graph

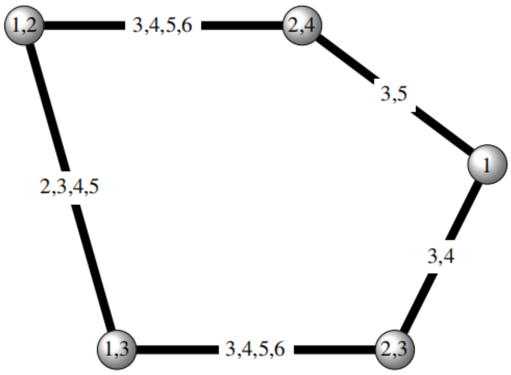
- Paths, stars, trees obtained from a star by replacing each edge of the star by a path.
- Kn for n = 1 n = 2

▪ **Sumset Labeling**

We can find sumset of two sets if both A and B are number sets.

S. Naduvath [39]

Definition: 15. Let G = G(V,E) be a connected graph with n vertices and m edges.Let A be a non-empty set of nonnegative integers and P(A) be the collection of the non-empty subsets of A i.e. power set of A. Then, an integer additive set-labelling or Sumset labeling of a graph G is an injective function f : V(G) → P(A) such that the induced function f^+ : V(G) → P(A) is defined by f^+(rs) = f(r) + f(s), for all rs ∈ E(G), where f(r) + f(s) is the sumset of the set-label the vertices r and s.



Types of Sumset Labeling of a Graph

- Modular Sumset Labeling.
- Weak Modular Sumset.
- Strong Modular Sumset.
- Maximal Modular Sumset Graph
- Exquisite Modular Sumset Graphs
- Arithmetic Sumset Labeling of Graphs
- K-uniform sumset labeling
  - **Power Mean Labeling of Graph**

Mercy P. [37]

Definition: 16. Let G = G(V,E) be a connected graph with n vertices and m edges. A graph G is said to be a Power Mean Graph if it is possible to label the vertices v ∈ V with distinct labels F(r) from {1,2,3,...,m+1}i.e. there exists an injective function F : V → {1,2,...,m}in such a way that when each edge e = rs is labeled with

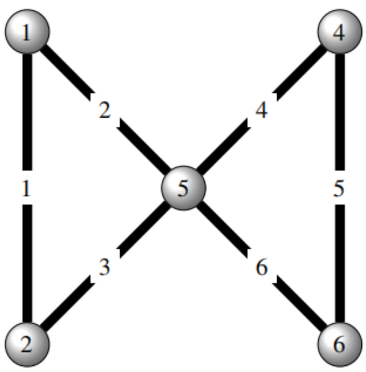
$$F(rs) = \left\lceil \left(F(r)^{F(s)} * F(s)^{F(r)}\right)^{\frac{1}{F(r)+F(s)}} \right\rceil$$

$$F(rs) = \left\lfloor \left(F(r)^{F(s)} * F(s)^{F(r)}\right)^{\frac{1}{F(r)+F(s)}} \right\rfloor$$

then the resulting edge labels are distinct and are from {1,2,3,...,m} .

F in called Power mean labeling of G.

Exa.



Examples of Power Mean Labeling of a Graph

- Tadpoles T(n, k) is a Power mean graph.

Result

- When any two cycles Cm and Cn are joined by a path Pn then the resultant graph is a Power mean graph
- T(Pm) , m ≥ 2.
- For n ≥ 2 Subdivision of any path is a Power mean graph.
- For n ≥ 3, a subdivision of any cycle Cn is a Power mean graph.

Applications of Graph Labeling Technique

Graph labeling is one of the crucial areas of the graph concept which has many applications in social community, verbal exchange community, circuit design, Database management, coding principle, radar, astronomy, Medical sciences and X-ray crystallography.

Conclusion

From these labeling techniques we observe that there is deep relationship between number theory and graph labeling techniques. Concept in number theory give rise to one labeling technique.

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