

MEASURE OF CONNECTIVITY (NETWORK) STRUCTURE: A STUDY

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ABSTRACT

The term 'structure' denotes the layout, geometry or network pattern of transportation facilities or systems (Garrison and Marble, 1963, p. 140). The expression, which may be used synonymously, imply a set of spatial relation between distinguishable elements of transportation network, in respect to each other and to the organised whole. By measuring such relations we can quantify the notion of structure that is we can describe the structure of transportation networks in mathematical terms. Measure are numerical symbols expressing relationship between given properties. The obvious advantage of measure is that the numerically expressed element of an ordered set can be put in a one-to-one relationship with an ordered finite class of real numbers.

Keywords: Structure, Layout, Geometry, Network Pattern, Transportation Networks.

Introduction

A study of relationship between the structure of transportation network and regional characteristics of areas demands exact measurement. There is need to make fine distinctions between individual network. It is also necessary to compare data relating to two sets of phenomena, each obtained under different circumstances. Hence, it is natural that we should take advantage of the abstract concept available in graph theory. Graph Theory as a branch of combinational which topology, provides us with an appropriate language suitable for movement and analysis of the structure of transportation networks.

Graphs defined as set of systematically organised point lines, are and lines, are similar visual representation of abstract concepts and relations. Seemingly abstract terms like 'Structure' gave visual and spatial meaning by applying basic graph theoretic terminology. Structure of transportation networks can be studied in two- or three-dimensional graph by using the theorems of graph theory. Fundamental concepts of graph theory similarly facilitate recognition of relation between seemingly non-correlated elements of transportation networks.

The study of networks in this topological sense began with Euler's 1736 paper on the seven bridges of the prussian city of Königsberg and continued with Capley's (1779) map colouring problem; but the first comprehensive treatment of network topology was not published until 1936 in Koning's *Theorie der endlichen und unendlichen Graphen*. The branch of topology dealing with elementary structure, which came to be called 'graph theory' has developed rapidly in the three decades since Koning's seminal work and is now summarized in a range of texts e.g. Berge (1962), Busacker and Saaty (1965), Flament (1963), Harary, Narmun and Cartwring (1965), Ore (1963). (1963). Garrison and Marble (1961) have designed a set of measures based on concepts of graph theory. Nystuen and Decey (1961) analysed functional connections between central places based upon communication flows in a network with the help of graph theoretic concepts. Garrison (1960) and Bhurton applied certain graph-theoretic measures to regional highway networks. These studies affirm the applicability of the analytical approach and graph-theoretic language to analysis of transportation networks. Measures of network structure are measures of relative quantity. There are two general categories of measure:

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- Measure of absolute quantity i.e. measure of length, capacity, physical design etc.
- Measure of relative quantity, i.e. measures designed as ratio between a property or physical quantity and another property of physical quantity. All measures used in the present study are either directly derived from graph theory or are inferred from graph theoretic concepts. Two general classes of measures will be found in the study.

Aggregate Measure of Network (Connectivity)

When the network is abstracted as graph the relation between the whole system and its individual system are established. By manipulating these elements, edge, node, subgraph, a series indices are evolved to analyse the structure of network.

Graph theoretic measures provide observed relationship of network into numerical and symbolic forms. Several indices have been developed within the graph theory to measure and reflect the topological attributes of the network. These indices reveal and translate into uniform and consistent measures of the geometrical structure of the network. Kansky (1963) has given two types of graph theoretic measures for evaluating the entire or aggregate network.

- **Non-Ratio Measures**

Measures describing transportation network in their aggregate are constructed predominantly as ratio between the whole system and its individual elements. Only two measures can be abstracted directly from graph theory of the network viz., (a) cyclomatic number and (b) associate number and the diameter.

- **Cyclomatic Number**

This is the measure of the number of circuits in the transportation system, or the number of links in the system excess to the number required to the vertices together in a minimal way. It gives one measure of relationship between actual edges and minimum edges i.e. cyclomatic number can be obtained by subtracting the 'θ' minimum from actual edges.

$$\mu = \text{actual edges} - \text{minimum edges (mm)}$$

$$\text{where, } \theta \text{ min} = (v-1)$$

$$\text{or } \theta \text{ min} = (v-g/p/x)$$

$$\text{Hence } \mu = (v-g/p/x)$$

$$\text{or } \mu = e-v + g/p/x$$

where θ = cyclomatic number

θ = number of edges or routes

v = number of non-connected subgraph.

$g/p/x$ = number of vertices or nodes

This index has two important properties for transportation studies:

- The cyclomatic number of a linear graph is equal to maximum number of independent circuits of graph G.
- In a connected graph G the cyclomatic number is equal to the maximum of fundamental circuits.

These two properties suggests this that measure of transportation network be may correlated with geographic characteristics of the the region high connected graph have higher cyclomatic numbers.

In the region under study, Shahpura, Manoharpur and Amarsar revenue circle have $\mu = 0$. The tehsil as a whole has $\mu = 0$ which indicates that the tehsil is moderately connected, which is the characteristics of a typical agricultural region in a plain area.

- **Associate Number**

This is the noted earlier that there were a number of ways in which the position of a single place on the system might be evaluated.

- The associated number is one indication of how accessible places are to be network. Associate numbers were computed by determining the longest distance for each place.
- An alternate method of measuring accessibility has been made following the method suggested by Shimmel and Katz.

$$T = SX + S^2X^2 + S^3X^3 + \dots S^nX^n + \dots$$

S is a Scalar, $0 < S < 1$, Measuring the effectiveness of a one-route connection, S^2 is the effectiveness of a path with two routes, S^2 is the effectiveness of an r length.

• **Ratio Measures**

The following ratio measures have been employed in structural analysis of the entire network :

- The Beta Index (β) (Kansky, 1963, p. 16) :

The beta index is the simplest form of the three measures which record the relation between two individual elements of a network. It gives the ratio of the number of linkage per place or node, this is linkage intensity. It is computed as :

$$\beta = e / v$$

where e is the number of edges, and

v is the number of vertices.

For Manoharpur β 1.225. Its logical basis is similar to that of the measures. Transportation networks with complicated structure will have high values of B, whereas network with a simplest structure will have low values. The whole scales of the Beta for planner graphs in form '0' to an upper limit to 'β'.

This index has the following valuable properties (i) Trees and disconnected graphs have value $\beta = 1$, (ii) A value of one is ascribed to any network which has only one circuit, and (iii) Higher values in the B index are produced by a complicated network structure, with a higher number of vertices.

In the region under study, B index vertices between 1.00 to 1.225. The highest B index observed in Manoharpur and the lowest for (1.00) Shahpura. For the region as a whole it comes to 1.135.

Table 3: Values for Different Revenue Circle by Various Indications

S. No.	Revenue Circle	Beta	Gamma	Alpha	Theta	Pi	Eta	Cyclomatic Number
1.	Amarsar	1.115	0.402	0.179	2.795	2.120	2.365	0
2.	Manoharpur	1.225	0.436	0.178	2.304	2.944	1.709	0
3.	Shahpura	1.00	0.360	0.212	3.105	2.510	2.185	0
Whole tehsil		1.135	0.388	0.146	2.710	2.867	2.344	0

• **The Gamma Index (γ)**

It is a ratio between the edges and vertices of a given transportation network. In other words, it is the ratio between the actual and the maximum possible number of edges in a graph. It is defined as :

For Planner graph

$$\text{or } \gamma = e / 3 (v-2)$$

e.g. for Manoharpur $\gamma = 38/3 (31-2)$

= 0.436 (Table 4.2).

The relationship between the number of nodes (v) and the maximum number of linkage (e) in a planner graph is away $3(v-2)$. The inclusion of one additional node to a network of more than two nodes increases the number of possible linkage by a value of 3. There is no inter section of linkage except at a node (Fig. 4.3). By applying the γ index a sequence of graph-theoretic value would be '0' between and '1'. It can also be expressed in percentage.

The highest value study region is associated with Manoharpur (0.336) and the lowest γ value for Shahpura (0.360) (Fig. 4.3). Whereas the γ value for other revenue circles show very little difference. The region as a whole has γ value of 0.388 (38.8%), which is even less than the values of individual revenue circles. It shows that the degree of connectivity of the region as a whole is lower than the revenue circles.

As the value of a index increase the degree of connectivity also increases. Like the β index the γ index also has a direct correlation with level of economic development; in the richer countries of the world, there are alternative and possible redundant links which improve the level of accessibility between nodes. The study region has very low values which indicates that the level of economy development is relatively low. It is because of the fact that it is primarily an agricultural region.

- **The Alpha Index (α)**

The alpha measure is an adjusted form of the cyclomatic number (μ). We have defined a minimally connected network as one in which there are no isolated nodes and the number of connecting linkage in one less than the number of nodes. If one linkage is removed, the network is divided into two completely separate parts.

Given a finite connected network in which the number of linkage is e and the number of node is v , the number of linkage is equal to one less than the number of node ($e = v-1$) only when the network is minimally connected, when circuit exist in the network the number of linkage needed for a minimally connected network ($v-1$) are subtracted from the actual number of linkage (e). This may be expressed as $e(v-1) = e-v + 1$. Here 1 refers to the existence of only one graph. It is a measure of the number of independent circuit in network.

The maximum number of independ circuit in a network is also a function of the number of nodes of nodes in the network and the number of linkage necessary for minimally connected between nodes. For a planner network the maximum number of edges is $3(v-2)$, thus the maximum number of circuits would be: $3(v-2) - (v-1) = \text{number of actual } 2v-5$. The index is a ratio measure of the circuits, given by $(e - v + 1)$, to the maximum number possible in a given network $(2v - 5)$.

For a Planner graph:

e.g. for Amarsar $\alpha =$

Values between '0' and '1' can be assigned to the network units by applying this index. It can be expressed in percentage also. This value indicates different degree of connectivity. The highest a value (.212) in the tehsil for Shahpura is misleading because there exists an assumed circuit formed by the connection of edges from Manoharpur in this revenue circle. The tehsil's whole α value is 0.146. The values for the region under study reveals that the area still has under developed road network. This has been because of the much smaller area of the revenue circles than the tehsil as a whole which has increased the number of an this index does not does not present assumed edges. Thus, it reveals that this index true picture of the network structure of the smaller areas.

- **The Eta Index (η)**

The ' η ' index expresses relations between the transportation network as a whole and its routes as individual element of network. It is given by the formula:

e.g. for Amarsar $\eta = 2.120$.

In the language of graph theory the η index is a ratio of the sum of all edges and all vertices to the observed number of edges of the network $(e + v/e)$. This ratio gains meaning if we substitute for the abstract form of the numerator a number representing the totality of the given transportation network. Thus we may write the ' η ' index as a ratio between the total kilometer age of the network and the observed number of edges, average edge length = km/e .

Where km is the total kilometer age of the network and e is observed number of edges. Because the numerator is a denominate number $(x \text{ km})^2$ the ratio is also a denominate number $(x \text{ kms})$. Hence, the η index, written km/e is a ratio indicating the length of an average edge in kilometres.

The eta index value for the region under study as a whole 2.344 and an average revenue circle level is 2.086 and at revenue circle level it varies between 1.709 to 2.365. In this index maximum variations in their values have been observed. The higher η values have been Amarsar (2.365), Shahpura (2.185), Manoharpur (1.709).

These variations are due to the varying length of the edges in relation to their number Amarsar, which is greater density of road and higher Eta values.

- **The Pi Index (π)**

The Pi index is a measure of relations between a transportation network as a whole and specific edges that of network. It is called the index for its logically. Similarity to the irrational number $\pi = 3.14159$. π is a number expressing the relationship between the circumference of a circle and its diameter. This index may be formulated as : $c = \pi d$

or $\pi = c/d$

where, c = circumference or total kilometer age of the network.

d = total kilometer age of the networks diameter

e.g., for Amarsar $\pi = 61.5/29$
 $= 2.120$.

Higher value of π index reflect higher degree of development of the network. In the region T index also shows great spatial variations. In different revenue circle it varies from 2.944 in Manoharpur to 2.510 in Shahpura and tehsil as a whole it is 2.867 values.

- **The Theta Index (θ)**

The index is a ratio of the network as a whole to its vertices. It may be written as :

$\theta = T/V$ in which T = The total traffic flows or the total volume of freight carried.

V = The observed number of vertices.

One also may express to totality of transportation networks in terms of the total kilometer age, as above, this index written as :

$\theta = Km/V$

where Km = the total kilometer age of the network.

V = the total observed number of vertices.

e.g. for Manoharpur : $\theta = 53.0/31$
 $= 1.709$

In the region under study e index values shows great spatial variations. In different revenue circles it varies between 2.795 for Amarsar, 2.304 for Manoharpur and 3.105 for Shahpura. The tehsil as a whole, it is 2.710.

Table 4: Relevant Data for Eta, Pi, Theta Indices

S.No.	Revenue Circle Distance	Whole Network Diameter*(km)	Distance (Km) of	No. of vertices	No. of edges
1.	Amarsar	61.5	29.0	26	29
2.	Manoharpur	53.0	18.0	31	38
3.	Shahpura	59.0	23.5	27	27
	Total :	173.5	60.5	84	94

- Instead of topological distance, the actual distance in km. between the two remotest nodes have been taken into account on the distance of diameter for the purpose of calculating the index only.

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