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# MULTI OBJECTIVE TIME DEPENDENT TRANSPORTATION PROBLEM FORMULATION AND THEIR SOLUTIONS

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## ABSTRACT

*Corporate and industries are fundamentally tackling with financial optimization such as cost minimization of non-economic items that are vibrant to the presence of their businesses. The transportation problem (TP) is one of the ultimate optimization problems in the branch of optimization or operations research (OR). The transportation problems mostly focus on the best possible way a product manufactured at different supply origins can be transported to a various demand destinations. The objective in a TP is to fully gratify the destination necessities within the functioning manufacture capacity constraints at the minimum feasible cost. The construction of transportation problem comprises a large number of transport routes from numerous supply origins to numerous demand destinations. The penalty i.e., the coefficients of the objective function can characterise transportation cost, time, profit, product defectiveness and so on. In this paper we have developed Multi objective Time dependent transportation problem formulation and their solutions by using fuzzy programming technique with two objective cost as a function of time and risk.*

***KEYWORDS***: *Multi- Objective, Transportation, Liner Regression, Fuzzy Programming, Cost, Time.*

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## Introduction

One important application of linear programming is in the area of physical distribution (transportation) of goods and services from several supply centres to several demand centres. It is easy to mathematically express a transportation problem in terms of LP problems, which can be solved by the simplex method. The structure of transportation problem involves a large number of shipping routes from several supply origins to several demand destinations. The objective is to determine the number of units of an item that should be shipped from an origin to a destination in order to satisfy the required quantity of goods or services available at each supply centre, at the minimum transportation cost or time. There are various types of transportation models and the simplest of them was presented by F L Hitchcock(1941).It was further developed by T C Koopmans(1949) and G B Dantzig (1951) [1]. The multi-objective transportation problem refers to a special class of linear programming problem in which the constraints are of equality type and all the objectives are conflicting each other. All the proposed methods to solve multi-objective linear programming problem generate a set of non-dominated or compromise solution. A variety of approaches, such as lexicographic goal programming approach, interactive algorithm, the step method, the utility function method have been developed by many researchers for the multi-objective linear programming problem. A.J Khan and D.K Das(2012) [2] used row maxima method to solve Multi-Objective transportation problem under fuzzy conditions. A.J Khan and D.K Das(2013) [3] used EMV(Expected Monetary value) approach to solve Multi-Objective transportation problem under fuzzy conditions.

Mohammad Asim Nomani, Irfan Ali and A. Ahmed [4] presented algorithm of proposed method in 2017. In proposed method they have used weighted sum method based on goal programming. In 2017 only, Sankar Kumar Roy, Gurupada Maity, Gerhard Wilhelm Weber and Sirma Zeynep Alparslan Gök [5] solved multi objective transportation problem by using conic scalarization approach with interval goal.

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Then, by using utility approach with goals Sankar Kumar Roy, Gurupada Maity and Gerhard-Wilhelm Weber [6] solved multi objective two stage grey transportation problem in 2017. By inspired from Zimmermann's fuzzy programming and the neutrosophic set terminology recently in 2018, Rizk M. Rizk-Allah, Aboul Ella Hassanien and Mohamed Elhoseny [7] proposed a model under neutrosophic environment. In this model for each objective functions, they considered three membership functions namely, truth membership, indeterminacy membership and falsity membership. Srikant Gupta, Irfan Ali and Aquil Ahmed [8] presented their study on multi objective capaciated transportation problem with uncertain supply and demand. They formulated deterministic form of the problem by using solution procedure of multi choice and fuzzy numbers. Then they used goal programming approach to solve fractional objective function. The linear interactive and discrete optimization (LINDO),general interactive optimizer(GINO) and TORA packages as well as many other commercial and academic packages are useful to find the solution of the transportation problem.

Corporate and industries are fundamentally tackling with financial optimization such as cost minimization of non-economic items that are vibrant to the presence of their businesses. The transportation problem (TP) is one of the ultimate optimization problems in the branch of optimization or operations research (OR). The transportation problems mostly focus on the best possible way a product manufactured at different supply origins can be transported to a various demand destinations. The objective in a TP is to fully gratify the destination necessities within the functioning manufacture capacity constraints at the minimum feasible cost. Whenever there is a physical effort of goods from the point of industrialist to the ultimate trades through a variety of frequencies of delivery (traders, dealers, distributors etc.), there is a necessity to minimize the cost of transportation to raise profit on sales. The construction of transportation problem comprises a large number of transport routes from numerous supply origins to numerous demand destinations. The penalty i.e., the coefficients of the objective function can characterise transportation cost, time, profit, product defectiveness and so on. The objective is to find the number of units dispatched from an origin to a destination in order to gratify the essential quantity of goods or services at each demand destination, within the limited quantity of goods or services presented at each supply origin at the minimum transportation cost or time. Transportation problem has been mostly used to solve decision-making problem in industrial organization, manufacturing system, developing service system, etc. In TP, generally two types of objectives (minimum and maximum) are measured. Minimization term is used for minimizing cost or time etc., while the maximization term is used for maximizing the overall profit or the overall quality, etc. In this paper we try to develop Multi objective Time dependent transportation problem formulation and their solutions.

**Data Collection of Multi Objective Time Dependent Transportation Problem**

This section represents the secondary data [9] collected by researchers. The data are as follows: A XYZ delivers its small packet of biscuits from four cities in India, namely Bengaluru (S1), Nashik (S2), Neemrana (S3) and Sitarganj (S4) to four cities in India, namely Kanpur (D1), Kolkata (D2), Surat (D3) and Hyderabad (D4).

The manger (Decision Maker) of a company wants to minimize a cost of transportation as well as minimize a risk of transportation for particular mode. They have used website of Indian post to collect the data for multi objective transportation problem formulation. They have added weight and dimensions of above mentioned product on postage calculator tool of that website and got the data for each sources and destinations Considering that the DM wants to transport a packet (containing 24 number of small packets) of Parle g biscuit with weight 2 kg, length 20 cm, width 18 cm and height 9 cm by express parcel mode of Indian post. Here, supply capacities of sources S1, S2, S3 and S4 are 20, 14, 27 and 19 respectively. Demand levels of destinations D1, D2, D3 and D4 are 17, 18, 24 and 11.

The time taken to transport a one big packet from post office of source cities to post office of destination cities is given in table 1.

**Table 1: Transportation Time (In Days)**

| | **Kanpur** | **Kolkata** | **Surat** | **Hyderabad** |
|---|---|---|---|---|
| Bengaluru | 5-6 | 2-4 | 5-6 | 2-4 |
| Nashik | 5-6 | 5-6 | 5-6 | 5-6 |
| Neemrana | 5-6 | 5-6 | 5-6 | 5-6 |
| Sitarganj | 5-6 | 5-6 | 5-6 | 5-6 |

The transportation cost to transport a one big packet from post office of source cities to post office of destination cities within a time interval shown in table 1 is given in table 2.

**Table 2: Transportation Cost (In Rs.)**

| | **Kanpur** | **Kolkata** | **Surat** | **Hyderabad** |
|---|---|---|---|---|
| Bengaluru | 165.20 | 146.32 | 165.20 | 146.32 |
| Nashik | 165.20 | 165.20 | 165.20 | 165.20 |
| Neemrana | 134.52 | 165.20 | 165.20 | 165.20 |
| Sitarganj | 134.52 | 165.20 | 165.20 | 165.20 |

Now, on the basis of data given in table 1 and table 2, for respective source and destination I break down the time interval in six parts and take cost data hypothetically which are shown in following tables.

**Table 3: Transportation Cost and Time for Bengaluru to Kanpur**

| | Kanpur | | | | | | |
|---|---|---|---|---|---|---|---|
| Bengaluru | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.75 | 165.45 | 165.2 | 165 | 164.75 | 164.6 |

**Table 4: Transportation Cost and Time for Bengaluru to Kanpur**

| | Kolkata | | | | | | |
|---|---|---|---|---|---|---|---|
| Bengaluru | Time(Days) | 2 | 3 | 4 | 5 | 6 | 7 |
| | Cost(Rs.) | 146.75 | 146.5 | 146.32 | 146.15 | 146 | 145.8 |

**Table: 5 Transportation cost and time for Bengaluru to Surat**

| | Surat | | | | | | |
|---|---|---|---|---|---|---|---|
| Bengaluru | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.65 | 165.4 | 165.2 | 165 | 164.95 | 164.7 |

**Table 6: Transportation Cost and Time for Bengaluru to Hyderabad**

| | Hyderabad | | | | | | |
|---|---|---|---|---|---|---|---|
| Bengaluru | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 146.72 | 146.45 | 146.32 | 146.18 | 146 | 145.8 |

**Table 7: Transportation Cost and Time for Nashik to Kanpur**

| | Kanpur | | | | | | |
|---|---|---|---|---|---|---|---|
| Nashik | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.8 | 165.5 | 165.2 | 165 | 164.75 | 164.6 |

**Table 8: Transportation Cost and Time for Nashik to Kolkata**

| | Kolkata | | | | | | |
|---|---|---|---|---|---|---|---|
| Nashik | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.75 | 165.4 | 165.2 | 165 | 164.8 | 164.6 |

**Table 9: Transportation Cost and Time for Nashik to Surat**

| | Surat | | | | | | |
|---|---|---|---|---|---|---|---|
| Nashik | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.7 | 165.4 | 165.2 | 165 | 164.75 | 164.6 |

**Table 10: Transportation Cost and Time for Nashik to Hyderabad**

| | Hyderabad | | | | | | |
|---|---|---|---|---|---|---|---|
| Nashik | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.7 | 165.45 | 165.2 | 165.05 | 164.9 | 164.7 |

**Table 11: Transportation cost and time for Neemrana to Kanpur**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Kanpur | | | | | | |
| Neemrana | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 134.95 | 134.75 | 134.52 | 134.3 | 134 | 133.8 |

**Table 12: Transportation cost and time for Neemrana to Kolkata**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Kolkata | | | | | | |
| Neemrana | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.75 | 165.45 | 165.2 | 165 | 164.8 | 164.65 |

**Table 13: Transportation cost and time for Neemrana to Surat**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Surat | | | | | | |
| Neemrana | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.7 | 165.45 | 165.2 | 165 | 164.75 | 164.5 |

**Table 14: Transportation Cost and Time for Neemrana to Hyderabad**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Hyderabad | | | | | | |
| Neemrana | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.85 | 165.5 | 165.2 | 165 | 164.7 | 164.45 |

**Table 15: Transportation Cost and Time for Sitarganj to Kanpur**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Kanpur | | | | | | |
| Sitarganj | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 134.95 | 134.75 | 134.52 | 134.3 | 134.1 | 133.85 |

**Table: 16 Transportation cost and time for Sitarganj to Kolkata**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Kolkata | | | | | | |
| Sitarganj | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.75 | 165.5 | 165.2 | 165 | 164.8 | 164.6 |

**Table 17: Transportation Cost and Time for Sitarganj to Surat**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Surat | | | | | | |
| Sitarganj | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.8 | 165.5 | 165.2 | 165 | 164.7 | 164.5 |

**Table 18: Transportation Cost and Time for Sitarganj to Hyderabad**

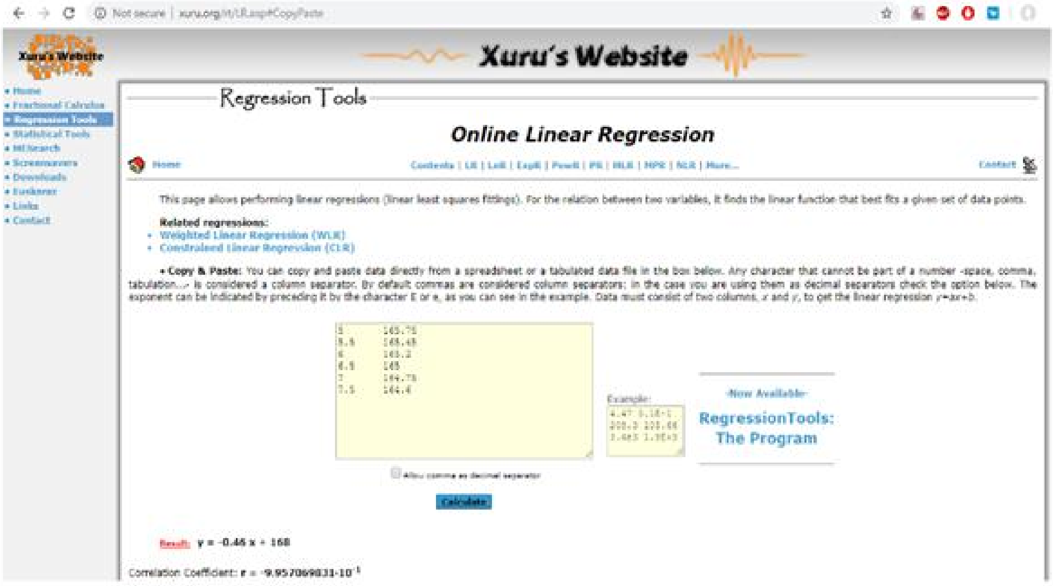
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Hyderabad | | | | | | |
| Sitarganj | Time(Days) | 5 | 5.5 | 6 | 6.5 | 7 | 7.5 |
| | Cost(Rs.) | 165.95 | 165.55 | 165.2 | 164.95 | 164.7 | 164.45 |

**Liner and Nonlinear Regression Trend of Transportation Cost**

To make cost as a function of time I have used linear regression as well as polynomial regression. For that I have used online regression tool of site Xuru.org. Numerous regressions are there few of them are listed below.

- Linear Regression (LR)
- Logarithmic Regression (LnR)
- Exponential Regression (ExpR)
- Power Regression (PowR)
- Polynomial Regression (PR)
- Multiple Linear Regression (MLR)
- Multiple Polynomial Regression (MPR)
- Nonlinear Regression (NLR)

Among them I have choose linear regression tool as well as polynomial regression tool to construct cost as a function of time. A layout of online regression tool is shown in figure:




To make cost as a function of time I have used linear regression and get following cost matrix.

**Table 19: Cost as a Function of Time with Linear Regression**

| | Kanpur | Kolkata | Surat | Hyderabad |
|---|---|---|---|---|
| Bengaluru | $C_{11}(t_{11})$ | $C_{12}(t_{12})$ | $C_{13}(t_{13})$ | $C_{14}(t_{14})$ |
| Nashik | $C_{21}(t_{21})$ | $C_{22}(t_{22})$ | $C_{23}(t_{23})$ | $C_{24}(t_{24})$ |
| Neemrana | $C_{31}(t_{31})$ | $C_{32}(t_{32})$ | $C_{33}(t_{33})$ | $C_{34}(t_{34})$ |
| Sitarganj | $C_{41}(t_{41})$ | $C_{42}(t_{42})$ | $C_{43}(t_{43})$ | $C_{44}(t_{44})$ |

Where,

$$C_{11}(t_{11}) = (-0.46(t_{11}) + 168), \qquad (t_{11}) \in [5, 7.5]$$

$$RSS = 0.008,\ R^2 = 0.9914323963$$

$$C_{12}(t_{12}) = (-1.834285714 \times 10^{-1}(t_{12}) + 147.0787619), \qquad (t_{12}) \in [2, 7]$$

$$RSS = 0.003527619048,\ R^2 = 0.9940445373$$

$$C_{13}(t_{13}) = (-0.36(t_{13}) + 167.4), \qquad (t_{13}) \in [5, 7.5]$$

$$RSS = 0.013,\ R^2 = 0.9775862069$$

$$C_{14}(t_{14}) = (-0.174(t_{14}) + 147.028), \qquad (t_{14}) \in [2, 7]$$

$$RSS = 0.00572, R^2 = 0.9893193913$$

$$C_{21}(t_{21}) = (-4.828571429 \times 10^{1}(t_{21}) + 168.1595238), \qquad (t_{21}) \in [5, 7.5]$$

$RSS = 0.01204761905, R^2 = 0.9883268931$

$C_{22}(t_{22}) = (-4.428571429 \times 10^{-1}(t_{22}) + 167.8928571), \quad (t_{22}) \in [5, 7.5]$

$RSS = 0.01071428571, R^2 = 0.9876670092$

$C_{23}(t_{23}) = (-4.342857143 \times 10^{-1}(t_{23}) + 167.8309524), \quad (t_{23}) \in [5, 7.5]$

$RSS = 0.006047619048, R^2 = 0.9928182654$

$C_{24}(t_{24}) = (-3.885714286 \times 10^{-1}(t_{24}) + 167.5952381), \quad (t_{24}) \in [5, 7.5]$

$RSS = 0.007761904762, R^2 = 0.9883861774$

$C_{31}(t_{31}) = (-4.422857143 \times 10^{-1}(t_{31}) + 137.1742857), \quad (t_{31}) \in [5, 7.5]$

$RSS = 0.00307047619, R^2 = 0.9968291124$

$C_{32}(t_{32}) = (-4.371428571 \times 10^{-1}(t_{32}) + 167.8738095), \quad (t_{32}) \in [5, 7.5]$

$RSS = 0.01104761905, R^2 = 0.9869580493$

$C_{33}(t_{33}) = (-4.742857143 \times 10^{-1}(t_{33}) + 168.0642857), \quad (t_{33}) \in [5, 7.5]$

$RSS = 0.0008571428571, R^2 = 0.9991298042$

$C_{34}(t_{34}) = (-5.485714286 \times 10^{-1}(t_{34}) + 168.5452381), \quad (t_{34}) \in [5, 7.5]$

$RSS = 0.006761904762, R^2 = 0.9948902483$

$C_{41}(t_{41}) = (-4.382857143 \times 10^{-1}(t_{41}) + 137.1509524), \quad (t_{41}) \in [5, 7.5]$

$RSS = 0.0006704761905, \ R^2 = 0.9992028421$

$C_{42}(t_{42}) = (-0.46(t_{42}) + 168.0166667), \quad (t_{42}) \in [5, 7.5]$

$RSS = 0.006333333333, \ R^2 = 0.9932051855$

$C_{43}(t_{43}) = (-0.52(t_{43}) + 168.3666667), \quad (t_{43}) \in [5, 7.5]$

$RSS = 0.005333333333, \ R^2 = 0.9955119215$

$C_{44}(t_{44}) = (-5.885714286 \times 10^{-1}(t_{44}) + 168.8119048), \quad (t_{44}) \in [5, 7.5]$

$RSS = 0.1776190476, \ R^2 = 0.9884161491$

To make cost as a function of time i have used polynomial regression and get following cost matrix.

**Multi Objective Transportation Problem Formulation**

Before formulating the model of multi objective transportation with cost as a function of time, first I have found solution for single objective only which is to minimize the transportation cost. For that I have considered three cases which are described as below.

**Case 1:** Cost matrix for first value of time interval (In days) for each sources and destinations.

$t_{11} = t_{13} = t_{21} = t_{22} = t_{23} = t_{24} = t_{31} = t_{32} = t_{33} = t_{34} = t_{41} = t_{42} = t_{43} = t_{44} = 5$ and $t_{12} = t_{14} = 2$

**Table 21: Transportation Cost using Linear Regression (In Rs.)**

| | **Kanpur** | **Kolkata** | **Surat** | **Hyderabad** | **Supply** |
|---|---|---|---|---|---|
| Bengaluru | 165.700 | 146.712 | 165.600 | 146.680 | 20 |
| Nashik | 165.745 | 165.679 | 165.655 | 165.652 | 14 |
| Neemrana | 134.974 | 165.688 | 165.693 | 165.802 | 27 |
| Sitarganj | 134.960 | 165.717 | 165.767 | 165.869 | 19 |
| Demand | 17 | 18 | 24 | 11 | |

**Case 2:** Cost matrix for middle value of time interval (In days) for each sources and destinations.

$t_{11} = t_{13} = t_{21} = t_{22} = t_{23} = t_{24} = t_{31} = t_{32} = t_{33} = t_{34} = t_{41} = t_{42} = t_{43} = t_{44} = 6$ and $t_{12} = t_{14} = 4$

**Table 23: Transportation Cost using Linear Regression (in Rs.)**

| | Kanpur | Kolkata | Surat | Hyderabad | Supply |
|---|---|---|---|---|---|
| Bengaluru | 165.240 | 146.345 | 165.240 | 146.332 | 20 |
| Nashik | 165.262 | 165.236 | 165.218 | 165.264 | 14 |
| Neemrana | 134.504 | 165.251 | 165.219 | 165.254 | 27 |
| Sitarganj | 134.521 | 165.257 | 165.247 | 165.280 | 19 |
| Demand | 17 | 18 | 24 | 11 | |

**Case 3:** Cost matrix for last value of time interval (In days) for each sources and destinations.

$t_{11} = t_{13} = t_{21} = t_{22} = t_{23} = t_{24} = t_{31} = t_{32} = t_{33} = t_{34} = t_{41} = t_{42} = t_{43} = t_{44} = 7.5$ and $t_{12} = t_{14} = 7$

**Table 25: Transportation Cost using Linear Regression (in Rs.)**

| | **Kanpur** | **Kolkata** | **Surat** | **Hyderabad** | **Supply** |
|---|---|---|---|---|---|
| Bengaluru | 164.550 | 145.795 | 164.700 | 145.810 | 20 |
| Nashik | 164.538 | 164.571 | 164.562 | 164.681 | 14 |
| Neemrana | 133.800 | 164.595 | 164.507 | 164.431 | 27 |
| Sitarganj | 133.864 | 164.567 | 164.467 | 164.398 | 19 |
| Demand | 17 | 18 | 24 | 11 | |

For multi objective transportation problem, here I have considered risk as a second objective. For that I have constructed risk matrix as shown in table 5. In which, I have considered risk value 1, 2, 3, 4 and 5 for those sources and destinations in which risk can be very low, low, neutral, high and very high.

**Table 27: Risk to Transporting a Unit Product for Particular Source and Destination**

| | **Kanpur** | **Kolkata** | **Surat** | **Hyderabad** | **Supply** |
|---|---|---|---|---|---|
| Bengaluru | 5 | 3 | 5 | 2 | 20 |
| Nashik | 3 | 4 | 1 | 1 | 14 |
| Neemrana | 4 | 5 | 2 | 3 | 27 |
| Sitarganj | 1 | 2 | 4 | 4 | 19 |
| Demand | 17 | 18 | 24 | 11 | |

By using the data of table 21 and table 27 the mathematical model of multi objective transportation problem with cost as a function of time is described as follows:

**Fuzzy Programming Technique to Solve Multi-Objective Problems**

Most of the entrepreneur now a day's do not have a aim of single objective but they wish to target multi objective i.e.' they not only try to minimize cost but try to minimize some recourse so that their business can grow in best of manner. In competitive world entrepreneur need to be aware of competition and should monopolized business. Their important objective could be to minimize risk using the same set of constraints. Such general multi objective linear programming problem can be defined as under [10,11]

Minimize $z_k = \sum_{i=1}^{i=n} c_i^k x_i, k = 1, 2, 3, 4....., r$

Subject to the constraints,

$$\sum_{i=1}^{n} a_i x_i (\leq, =, \geq) B_j,\ j = 1, 2, 3......m\ ,$$

$$x_i \geq 0\ .$$

In fuzzy programming technique following procedure applied to solve the multi objective optimization problem [12]:

The formulated multi objective linear programming problem first solve by using single objective function and derive optimal solution say $f_1(x_1, x_2, x_3..........x_n)$ for first objective $z_{11}$ and then obtain other objective value with the same solution say $z_{21}\ z_{31}\ z_{41} \ldots z_{k1}$. Procedure repeats same for $z_2........z_r$ objectives.

**Step 2**: Corresponding to above data we can construct a pay off matrix which can give various alternate optimal value.

**Table 28: Pay-off Matrix for MOLPP**

| | $Z_1$ | $Z_2$ | ......... | $Z_r$ |
|---|---|---|---|---|
| $f_1(x_1, x_2, x_3..........x_n)$ | $Z_{11}$ | $Z_{21}$ | ......... | $Z_{r1}$ |
| $f_2(x_1, x_2, x_3..........x_n)$ | $Z_{12}$ | $Z_{22}$ | ......... | $Z_{r2}$ |
| ...... | | | ......... | |
| $f_n(x_1, x_2, x_3..........x_n)$ | $Z_{1n}$ | $Z_{2n}$ | ......... | $Z_{rn}$ |

Here,

$z_{ki}$ : indicated optimal solution of 'k[th] objective using solution of 'i[th] objective, $k = 1, 2, 3, 4....., r$ and $i = 1, 2, 3......n$ . Or Find out the positive ideal solution (PIS) and negative ideal solution (NIS) for each objective function of the model. Now, by using pay-off matrix or positive ideal solution (PIS) and negative ideal solution (NIS) define a membership function $\mu(Z_k)$ for the $k^{th}$ objective function. Here two different membership function are utilized to find efficient solution of this multi-objective resource allocation problem and by using this membership function convert the MOLPP into the following model

**Model**

Maximum a ,

Subject to the constraints

$$a \leq «\left(Z_k\right),$$

$$\sum_{i=1}^{m} a_i x_i (\leq, =, \geq) B_j,\ j = 1, 2, 3......n$$

$$x_i \geq 0\ ,$$

When we utilize Fuzzy linear membership function [12] then model structure is as follows

**Model**

Maximum a ,

Subject to the constraints

$z_k + a(U_k - L_k) \le U_k$,

$\sum_{i=1}^{m} a_i x_i (\le, =, \ge) B_j,\ j = 1, 2, 3......n$

$x_i \ge 0$.

Solution of this model will give you an efficient solution

**Algorithm to Solve Multi-Objective Linear Programming Problem**

**Input:** Parameters: $(Z_1, Z_2, ..., Z_k, n)$

**Output : S**olution of multi-objective programming problem

Solve multi-objective programming problem ( $Z_k \downarrow, X \uparrow$ )

**begin**

**read:** problem

**while** problem = multi-objective programming problem do

**for** k=1 to m do

enter matrix $Z_k$

**end**

**-| determine pay-off matrix**

Or

**-| the positive ideal solution and negative ideal solution for each objective.**

**for** k=1 to m do

$z_{ij}^{\mathrm{PIS}} = \min\left(z_i\right)^0$

Under given constraints

**end**

**for** k=1 to m do

$z_{ij}^{\mathrm{NIS}} = \max\left(z_i\right)^0$

Under given constraints

**end**

**- find single objective optimization models under given constraints from multi-objective optimization models.**

**for**k=1 to m do

$\max a$

Subject to the constraints:

$a \le «_{Z_{ij}}^{E}\left(x\right)$

Under given constraints

**End**

**|-** find the solution SOPs using Lingo software.

**Multi Objective Time Dependent Transportation Problem Solutions by Fuzzy Programming Technique**

This section discussed formulation and solution of Time dependent transportation problem by fuzzy programming technique

Using data of table 21 and table 27 we get

Min $Z_1$ $=165.700*x_{11}+146.712*x_{12}+165.600*x_{13}+146.680*x_{14}+165.745*x_{21}+165.679*x_{22}+165.655*x_{23}+165.652*x_{24}+134.974*x_{31}+165.688*x_{32}+165.693*x_{33}+165.802*x_{34}+134.960*x_{41}+165.717*x_{42}+165.767*x_{43}+165.869*x_{44}$.

Min $Z_2 = 5*x_{11} +3*x_{12} +5*x_{13} +2*x_{14} +3*x_{21} +4*x_{22} +x_{23} +x_{24} +4*x_{31} +5*x_{32} +2*x_{33} +3*x_{34} +x_{41}+2*x_{42}+4*x_{43}+4*x_{44}$.

Subject to the constraints

$x_{11}+x_{12}+x_{13}+x_{14} = 20$

$x_{21}+x_{22}+x_{23}+x_{24} = 14$

$x_{31}+x_{32}+x_{33}+x_{34} = 27$

$x_{41}+x_{42}+x_{43}+x_{44} = 19$

$x_{11}+x_{21}+x_{31}+x_{41} = 17$

$x_{12}+x_{22}+x_{32}+x_{42} = 18$

$x_{13}+x_{23}+x_{33}+x_{43} = 24$

$x_{14}+x_{24}+x_{34}+x_{44} = 11$

Where $x_{ij} \geq 0$ for all i and j

**Solution of the Model**

Steps to solve multi objective transportation problem using fuzzy programming approach.

PIS and NIS value of first objective function is given by

PIS = 10,695.50, NIS = 13,258.76

PIS and NIS value of second objective function is given by

PIS = 125, NIS = 354

Hence,

$U_1$= 13,258.76, $L_1$=10,695.50, $U_2$= 354, $L_2$=125

$U_1 - L_1$= 2563.26

$U_2 - L_2$ = 229

Defining membership function by putting values of $L_r$ and $U_r$ for each objective functions.

($b_1$) Linear membership function:

$$\text{«}_1(X) = \begin{cases} 1, & if\ Z_1 \leq 10695.50 \\ 1-\dfrac{Z_1-10695.50}{13258.76-10695.50}, & if\ 10695.50 < Z_1 < 13258.76 \\ 0, & if\ Z_1 \geq 13258.76 \end{cases}$$

$$\text{«}_2(X) = \begin{cases} 1, & if\ Z_2 \leq 125 \\ 1-\dfrac{Z_2-125}{354-125}, & if\ 125 < Z_2 < 354 \\ 0, & if\ Z_2 \geq 354 \end{cases}$$

**Step 4:** For each membership functions defining crisp model using an auxiliary variable h.

($b_1$) Linear membership function:

Maximize $h$ ;

Subject to the constraints $Z_1\left(X\right)+h(13258.76-10695.50)\leq 13258.76$;

$$Z_1 = 165.700X_{11} + 146.712X_{12} + 165.600X_{13} + 146.680X_{14} +$$
$$165.745X_{21} + 165.679X_{22} + 165.655X_{23} + 165.652X_{24} +$$
$$134.974X_{31} + 165.688X_{32} + 165.693X_{33} + 165.802X_{34} +$$
$$134.960X_{41} + 165.717X_{42} + 165.767X_{43} + 165.869X_{44};$$
$$Z_2\left(X\right)+h(354-125)\leq 354$$
$$Z_2 = 5X_{11} + 3X_{12} + 5X_{13} + 2X_{14} + 3X_{21} + 4X_{22} + 1X_{23} + 1X_{24} +$$
$$4X_{31} + 5X_{32} + 2X_{33} + 3X_{34} + 1X_{41} + 2X_{42} + 4X_{43} + 4X_{44};$$
$$X_{11} + X_{12} + X_{13} + X_{14} \leq 20;$$
$$X_{21} + X_{22} + X_{23} + X_{24} \leq 14;$$
$$X_{31} + X_{32} + X_{33} + X_{34} \leq 27;$$
$$X_{41} + X_{42} + X_{43} + X_{44} \leq 19;$$
$$X_{11} + X_{21} + X_{31} + X_{41} \geq 17;$$
$$X_{12} + X_{22} + X_{32} + X_{42} \geq 18;$$
$$X_{13} + X_{23} + X_{33} + X_{43} \geq 24;$$
$$X_{14} + X_{24} + X_{34} + X_{44} \geq 11;$$
$$X_{ij} \geq 0, \forall i, j;$$
$$h \geq 0.$$

When we solve this problem with computational software like LINGO then the solution of the model is as follows:The allocations are,

$$X_{11} = 0, X_{12} = 16, X_{13} = 0, X_{14} = 4,$$
$$X_{21} = 0, X_{22} = 0, X_{23} = 7, X_{24} = 7,$$
$$X_{31} = 0, X_{32} = 0, X_{33} = 17, X_{34} = 0,$$
$$X_{41} = 17, X_{42} = 2, X_{43} = 0, X_{44} = 0.$$

The values of objective functions are as follows:

$Z_1$ = 10695.80, $Z_2$ = 125

Using these allocations we have $Z_1$=10695.80, $Z_2$= 125 with degree of satisfaction=0.9998845

The table given below shows the comparison of the given transportation problem with other approaches

| Method | Objective value |
|---|---|
| Goal Programming approach [13] | $Z_1$= 12353.030,$Z_2$=148. |
| Fuzzy Programming approach | $Z_1$=10695.80, $Z_2$= 125 |

**Using Data of Table 23 and Table 27**

By using the data of table 4.23 and table 4.27 the mathematical model of multi objective transportation problem with cost as a function of time is described as follows:

**Model**

Min$Z_1$ $=165.240*x_{11}+146.345*x_{12}+165.240*x_{13}+146.332*x_{14}+165.262*x_{21}+165.236*x_{22}+165.218*x_{23}+165.264*x_{24}+134.504*x_{31}+165.251*x_{32}+165.219*x_{33}+165.254*x_{34}+134.521*x_{41}+165.257*x_{42}+165.247*x_{43}+165.280*x_{44}$.

Min $Z_2 = 5*x_{11} +3*x_{12} +5*x_{13} +2*x_{14} +3*x_{21} +4*x_{22} +x_{23} +x_{24} +4*x_{31} +5*x_{32} +2*x_{33} +3*x_{34} +x_{41}+2*x_{42}+4*x_{43}+4*x_{44}$.

Subject to the constraints

$x_{11}+x_{12}+x_{13}+x_{14} = 20$

$x_{21}+x_{22}+x_{23}+x_{24} = 14$

$x_{31}+x_{32}+x_{33}+x_{34} = 27$

$x_{41}+x_{42}+x_{43}+x_{44} = 19$

$x_{11}+x_{21}+x_{31}+x_{41} = 17$

$x_{12}+x_{22}+x_{32}+x_{42} = 18$

$x_{13}+x_{23}+x_{33}+x_{43} = 24$

$x_{14}+x_{24}+x_{34}+x_{44} = 11$

Where $x_{ij} \geq 0$ for all i and j

Solution of the model by Fuzzy programming approach

Steps to solve multi objective transportation problem using fuzzy programming approach.

**Step 1:** Solution of each objective functions.

| | **Minimum** | **Maximum** |
|---|---|---|
| Cost (In Rs.) | 10,665.85 | 13,220.36 |
| Risk | 125 | 354 |

**Step 2:** Defining $L_r$ and $U_r$ to the minimum and maximum value of each objective functions respectively.

For cost $L_1 = 10,665.85 \ and \ U_1 = 13,220.36$

For risk $L_2 = 125 \ and \ U_2 = 354$

Step: (3) Defining membership function by putting values of $L_r$ and $U_r$ for each objective functions.

($b_1$) Linear membership function:

$$«_1(X) = \begin{cases} 1, \ if \ \ Z_1 \leq 10665.85 \\ 1 - \dfrac{Z_1 - 10665.85}{13220.36 - 10665.85}, \ \ if \ 10665.85 < Z_1 < 13220.36 \\ 0, \ if \ \ Z_1 \geq 13220.36 \end{cases}$$

$$\mu_2(X)=\begin{cases}1, \; if \;\; Z_2 \le 125 \\ 1-\dfrac{Z_2-125}{354-125}, \;\; if \;\; 125 < Z_2 < 354 \\ 0, \;\; if \; Z_2 \ge 354\end{cases}$$

**Step 4:** For each membership functions defining crisp model using an auxiliary variable h.

($b_1$) Linear membership function:

Maximize $h$ ;

Subject to the constraints $Z_1(X)+h(13220.36-10665.85)\le 13220.36$ ;

$$\begin{aligned}Z_1 = {} & 165.240X_{11}+146.345X_{12}+165.240X_{13}+146.332X_{14}+ \\ & 165.262X_{21}+165.236X_{22}+165.218X_{23}+165.264X_{24}+ \\ & 134.504X_{31}+165.251X_{32}+165.219X_{33}+165.254X_{34}+ \\ & 134.521X_{41}+165.257X_{42}+165.247X_{43}+165.280X_{44};\end{aligned}$$

$$Z_2(X)+h(354-125)\le 354$$

$$\begin{aligned}Z_2 = {} & 5X_{11}+3X_{12}+5X_{13}+2X_{14}+3X_{21}+4X_{22}+1X_{23}+1X_{24}+ \\ & 4X_{31}+5X_{32}+2X_{33}+3X_{34}+1X_{41}+2X_{42}+4X_{43}+4X_{44};\end{aligned}$$

$$X_{11}+X_{12}+X_{13}+X_{14}\le 20;$$

$$X_{21}+X_{22}+X_{23}+X_{24}\le 14;$$

$$X_{31}+X_{32}+X_{33}+X_{34}\le 27;$$

$$X_{41}+X_{42}+X_{43}+X_{44}\le 19;$$

$$X_{11}+X_{21}+X_{31}+X_{41}\ge 17;$$

$$X_{12}+X_{22}+X_{32}+X_{42}\ge 18;$$

$$X_{13}+X_{23}+X_{33}+X_{43}\ge 24;$$

$$X_{14}+X_{24}+X_{34}+X_{44}\ge 11;$$

$$X_{ij}\ge 0, \forall i, j;$$

$$h\ge 0.$$

When we solve this problem with computational software like LINGO then the solution of the model is as follows: The allocations are ,

$X_{11}=0, X_{12}=16, X_{13}=0, X_{14}=4,$

$X_{21}=0, X_{22}=0, X_{23}=7, X_{24}=7,$

$X_{31}=0, X_{32}=0, X_{33}=17, X_{34}=0,$

$X_{41}=17, X_{42}=2, X_{43}=0, X_{44}=0.$

The values of objective functions are as follows

$Z_1$ = 10666.32, $Z_2$ = 125

Using these allocations we have $Z_1$=10666.32, $Z_2$= 125 with degree of satisfaction=0.9998176

The table given below shows the comparison of the given transportation problem with other approaches

| Method | Objective value |
|---|---|
| Goal Programming approach [13] | $Z_1$= 10666.320,$Z_2$=125. |
| Fuzzy Programming approach | $Z_1$=10666.32, $Z_2$= 125 |

**Using Data of Table 25 and Table 27**

By using the data of table 4.25 and table 4.27 the mathematical model of multi objective transportation problem with cost as a function of time is described as follows:

**Model**

Min$Z_1$ =164.550*$x_{11}$+145.795*$x_{12}$+164.700*$x_{13}$+145.810*$x_{14}$+164.538*$x_{21}$+164.571*$x_{22}$+164.562*$x_{23}$+164.681*$x_{24}$+133.800*$x_{31}$+164.595*$x_{32}$+164.507*$x_{33}$+164.431*$x_{34}$+133.864*$x_{41}$+164.567*$x_{42}$+164.467*$x_{43}$+164.398*$x_{44}$.

Subject to the constraints

$x_{11}+x_{12}+x_{13}+x_{14} = 20$

$x_{21}+x_{22}+x_{23}+x_{24} = 14$

$x_{31}+x_{32}+x_{33}+x_{34} = 27$

$x_{41}+x_{42}+x_{43}+x_{44} = 19$

$x_{11}+x_{21}+x_{31}+x_{41} = 17$

$x_{12}+x_{22}+x_{32}+x_{42} = 18$

$x_{13}+x_{23}+x_{33}+x_{43} = 24$

$x_{14}+x_{24}+x_{34}+x_{44} = 11$

Where $x_{ij} \geq 0$ for all i and j

Solution of the model by Fuzzy programming approach

Steps to solve multi objective transportation problem using fuzzy programming approach.

**Step 1:** Solution of each objective functions.

| | Minimum | Maximum |
|---|---|---|
| Cost (In Rs.) | 10,618.04 | 13,166.26 |
| Risk | 125 | 354 |

**Step 2:** Defining $L_r$ and $U_r$ to the minimum and maximum value of each objective functions respectively.

For cost $L_1 = 10,618.04$ *and* $U_1 = 13,166.26$

For risk $L_2 = 125$ *and* $U_2 = 354$

Step: (3) Defining membership function by putting values of $L_r$ and $U_r$ for each objective functions.

($b_1$) Linear membership function:

$$\text{«}_1(\mathrm{X}) = \begin{cases} 1, \ \ if \ \ Z_1 \le 10618.04 \\ 1 - \dfrac{Z_1 - 10618.04}{13166.26 - 10618.04}, \ \ if \ \ 10618.04 < Z_1 < 13166.26 \\ 0, \ \ if \ \ Z_1 \ge 13166.26 \end{cases}$$

$$\text{«}_2(\mathrm{X}) = \begin{cases} 1, \ \ if \ \ Z_2 \le 125 \\ 1 - \dfrac{Z_2 - 125}{354 - 125}, \ \ if \ \ 125 < Z_2 < 354 \\ 0, \ \ if \ Z_2 \ge 354 \end{cases}$$

Step 4: For each membership functions defining crisp model using an auxiliary variable h.

($b_1$) Linear membership function:

Maximize $h$ ;

Subject to the constraints $Z_1\left(X\right) + h(13166.26 - 10618.04) \le 13166.26$ ;

$$\begin{aligned} Z_1 = {} & 164.550X_{11} + 145.795X_{12} + 164.700X_{13} + 145.810X_{14} + \\ & 164.538X_{21} + 164.571X_{22} + 164.562X_{23} + 164.681X_{24} + \\ & 133.800X_{31} + 164.595X_{32} + 164.507X_{33} + 164.431X_{34} + \\ & 133.864X_{41} + 164.567X_{42} + 164.467X_{43} + 164.398X_{44}; \end{aligned}$$

$$Z_2\left(X\right) + h(354 - 125) \le 354$$

$$\begin{aligned} Z_2 = {} & 5X_{11} + 3X_{12} + 5X_{13} + 2X_{14} + 3X_{21} + 4X_{22} + 1X_{23} + 1X_{24} + \\ & 4X_{31} + 5X_{32} + 2X_{33} + 3X_{34} + 1X_{41} + 2X_{42} + 4X_{43} + 4X_{44}; \end{aligned}$$

$$X_{11} + X_{12} + X_{13} + X_{14} \le 20;$$

$$X_{21} + X_{22} + X_{23} + X_{24} \le 14;$$

$$X_{31} + X_{32} + X_{33} + X_{34} \le 27;$$

$$X_{41} + X_{42} + X_{43} + X_{44} \le 19;$$

$$X_{11} + X_{21} + X_{31} + X_{41} \ge 17;$$

$$X_{12} + X_{22} + X_{32} + X_{42} \ge 18;$$

$$X_{13} + X_{23} + X_{33} + X_{43} \ge 24;$$

$$X_{14} + X_{24} + X_{34} + X_{44} \ge 11;$$

$$X_{ij} \ge 0, \forall i, j;$$

$$h \ge 0.$$

When we solve this problem with computational software like LINGO then the solution of the model is as follows:

The allocations are,

$X_{11}=0, X_{12}=16, X_{13}=0, X_{14}=4,$

$X_{21}=0, X_{22}=0, X_{23}=7, X_{24}=7,$

$X_{31}=0, X_{32}=0, X_{33}=17, X_{34}=0,$

$X_{41}=17, X_{42}=2, X_{43}=0, X_{44}=0.$

The values of objective functions are as follows

$Z_1$ = 10622.10, $Z_2$ = 125

Using these allocations we have $Z_1$=10622.10, $Z_2$= 125 with degree of satisfaction=0.9984059

The table given below shows the comparison of the given transportation problem with other approaches

| Method | Objective value |
|---|---|
| Goal Programming approach [13] | $Z_1$= 10622.930,$Z_2$=125. |
| Fuzzy Programming approach | $Z_1$=10622.10, $Z_2$= 125 |

**Results and Discussion**

| | $h$ | *Minimum of $Z_1$* | *Minimum of $Z_2$* | Solutions |
|---|---|---|---|---|
| Model 5.1 | 0.9998845 | 10695.80 | 125 | $X_{11}=0, X_{12}=16, X_{13}=0, X_{14}=4,$<br>$X_{21}=0, X_{22}=0, X_{23}=7, X_{24}=7,$<br>$X_{31}=0, X_{32}=0, X_{33}=17, X_{34}=0,$<br>$X_{41}=17, X_{42}=2, X_{43}=0, X_{44}=0.$ |
| Model 5.3 | 0.9998176 | 10666.32 | 125 | $X_{11}=0, X_{12}=16, X_{13}=0, X_{14}=4,$<br>$X_{21}=0, X_{22}=0, X_{23}=7, X_{24}=7,$<br>$X_{31}=0, X_{32}=0, X_{33}=17, X_{34}=0,$<br>$X_{41}=17, X_{42}=2, X_{43}=0, X_{44}=0.$ |
| Model 5.5 | 0.9984059 | 10622.10 | 125 | $X_{11}=0, X_{12}=16, X_{13}=0, X_{14}=4,$<br>$X_{21}=0, X_{22}=0, X_{23}=7, X_{24}=7,$<br>$X_{31}=0, X_{32}=0, X_{33}=17, X_{34}=0,$<br>$X_{41}=17, X_{42}=2, X_{43}=0, X_{44}=0.$ |

Solutions of multi objective transportation problems with cost as a function of time and risk is noted in table above, which shows that for different value of time, objective function of cost is different and objective function of the risk remain same for specific $t_{11}=t_{13}=t_{21}=t_{22}=t_{23}=t_{24}=t_{31}=t_{32}=t_{33}=t_{34}=t_{41}=t_{42}=t_{43}=t_{44}=5$ and $t_{12}=t_{14}=2$ When we form a multi objective transportation problem then the values of the objective functions are $Z_1$=10695.80, $Z_2$= 125 with degree of satisfaction=0.9998845. For specific $t_{11}=t_{13}=t_{21}=t_{22}=t_{23}=t_{24}=t_{31}=t_{32}=t_{33}=t_{34}=t_{41}=t_{42}=t_{43}=t_{44}=6$ and $t_{12}=t_{14}=4$. When we form a multi objective transportation problem then the values of the objective functions are $Z_1$=10666.32, $Z_2$= 125 with degree of satisfaction=0.9998176. For specific $t_{11}=t_{13}=t_{21}=t_{22}=t_{23}=t_{24}=t_{31}=t_{32}=t_{33}=t_{34}=t_{41}=t_{42}=t_{43}=t_{44}=7.5$

When we form a multi objective transportation problem then the values of the objective Z1=10622.10, Z2= 125 with degree of satisfaction=0.9984059. functions are

**Conclusion**

This paper discussed time dependent multi objective transportation problem and their solutions by fuzzy programming technique. Such kind of solution is useful in technology based today's atmosphere. With time dependent multi objective transportation problem we minimize the cost and minimize risk with respect to time.

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