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ANALYTIC STUDY OF WAITING TIME OF SERVICE IN FOUR SERVER HIERARCHICAL STRUCTURED FEEDBACK QUEUING SYSTEM WITH REVISIT AT MOST ONCE TO ANY OF THE SERVERS

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ABSTRACT

The present paper deals with the study of mathematical and graphical study of waiting time of service of a customer. The queuing system has four servers for the service of customers in hierarchical order. A customer, after getting service from a server may leave the system or may go for further service to the higher order server depending upon the need of service but cannot go for service more than two times. The arrival and service pattern are assumed to follow the Poisson process. The waiting time of customer for the service has been calculated from the mean queue length obtained by using generating function technique.

Keywords: *Feedback, Queuing System, Poisson Process, Four Server, Waiting Time.*

Introduction

In the Queueing theory we deal with the study of optimizing the waiting time of service. Generally, queues are formed when the supply of service does not balance with the demand of service. We face queueing problem everywhere in our day-to-day life such as in the banks, post offices, hospitals, airports, highway tolls etc. Many authors did a lot of work on queueing theory.

Mean Queue length of a feedback queueing system having three service channels wherein a customer may go forward/back to any service channel can be found. But there is no restriction on the number of such movements[1]. Mean queue length of the feedback queueing system changes with respect to different queueing characteristics keeping the other parameters constant which comprises of three servers linked in series hierarchically [2]. The sensitivity analysis of a multiple-server queuing system with reverse balking and reneging is carried out by obtaining the performance measures like average system size, average rate of reneging, and average rate of reverse balking are obtained[3].Designing the queue size in terms of the steady-state probabilities is a more reasonable and rational method than the expected queue length[4].A simple and direct approach have been provided to determine quality control policy and optimization of the Markovian feedback customers that has single-server and limited system capacity under steady-state situation adding the concepts of balking and retention of reneged customers[5]. After the completion of servicing in the high-speed server of a system

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with heterogeneous servers, MMPP flow, and instant feedback, primary calls according to the Bernoulli scheme either leave the system or immediately require re-servicing. Repeated (feedback) calls are served in a low-speed server, and after the service is completed, feedback calls can be repeated many times[6].System size along with other performance measures of a multi server Markovian queuing model can be obtained under the assumption that customers are state aware so that their impatience is state dependent [7]. A stochastic queuing model developed and solved for the system facing issues on the impatient behaviour of customers can help firms measure their performance well in advance and device strategies for effective management of the system[8]. Transient state probabilities for exact number of arrivals and departures from the two-state retrial queueing systems applicable in computer systems with feedback having two identical parallel servers, can be obtained when both, one or none of the servers is busy [9]. Mean queue length and other queueing characteristics of a system of complex feedback queue model containing three subsystems; one is comprised with two biserial service channels and other with two parallel service channels can be obtained in which both the subsystems are commonly connected with a central service channel[10].An extensive review of studies addressing queueing-related problems explicitly related to patient scheduling and queueing in emergency departments is provided and discussed scientific methodologies used to analyse and optimize algorithms, priority models, queueing models, simulation, and statistical approaches [11].An M/M/1 retrial queue under differentiated vacations and Bernoulli feedback policy is analysed in which on receiving the service, if the customer is not satisfied, then he may join the retrial group again with some probability and demand for service or may leave the system with the complementary probability[12].Some performance measures such as mean queue length of the system, waiting time etc. of a single server batch service queue model with feedback and second optional service under a transient and steady state environment can be analyzed [13]. Mean queue length of a four-server hierarchical structured queuing system with feedback and revisit of customer at most once to any of the servers increases and decreases with respect to different parameters [14].

No author worked on mathematical and graphical study of waiting time of a four-server hierarchical structured queuing system with feedback and revisit of customers at most once to any of the servers. Keeping this in view, here we have calculated the waiting time of the whole queuing system from the mean queue length and analysed graphically how the waiting time changes with respect to different queueing characteristics.

Notations

λ: Mean Arrival rate at 1st server (S_1)

μ_1: service rate of 1st server (S_1)

μ_2: service rate of 2nd server.

μ_3: service rate of 3rd server.

μ_4: service rate of 4th server

a_1: the probability of customer leaving 1st server 1st time.

a_2: the probability of customer leaving 1st server 2nd time.

b_1: the probability of customer leaving 2nd server 1st time.

b_2: the probability of customer leaving 2nd server 2nd time.

c_1: the probability of customer leaving 3rd server 1st time.

c_2: the probability of customer leaving 3rd server 2nd time.

d_1: the probability of customer leaving 4th server 1st time.

d_2: the probability of customer leaving 4th server 2nd time.

p_{12}: the probability of customer going from 1st to 2nd server 1st time.

p_1: the probability of exit of customer from 1st server 1st time.

p'_{12}: the probability of customer going from 1st to 2nd server 2nd time.

p'_1: the probability of exit of customer from 1st server 2nd time.

p_2: the probability of exit of customer from 2nd server 1st time.

p_{23}: the probability of customer going from 2nd to 3rd server 1st time.

p_{21}: the probability of customer going from 2nd to 1st server 1st time.

p_2': the probability of exit of customer from 2nd server 2nd time.

p_{23}': the probability of customer going from 2nd to 3rd server 2nd time.

p'_{21}: the probability of customer going from 2nd to 1st server 2nd time.

p_3: the probability of exit of customer from 3rd server 1st time.

p_{31}: the probability of customer going from 3rd to 1st server 1st time.

p_{32}: the probability of customer going from 3rd to 2nd server 1st time.

p_{34}: the probability of customer going from 3rd to 4th server 1st time

p_3': the probability of exit of customer from 3rd server 2nd time.

p'_{31}: the probability of customer going from 3rd to 1st server 2nd time.

p'_{32}: the probability of customer going from 3rd to 2nd server 2nd time.

p'_{34}: the probability of customer going from 3rd to 4th server 2nd time.

p_4: the probability of exit of customer from 4th server 1st time.

p'_4: the probability of exit of customer from 4th server 2nd time.

p_{41}: the probability of exit of customer from 4th to 1st server 1st time.

p_{42}: the probability of customer going from 4th to 2nd server 1st time.

p_{43}: the probability of customer going from 4th to 3rd server 1st time.

According to the model hypothesis we have:

$a_1p_1 + a_1p_{12} + a_2p_1' + a_2p'_{12} = 1$

$b_1p_2 + b_1p_{23} + b_2p_{21} + b_2p'_2 + b_2p'_{23} = 1;$

$c_1p_3 + c_1p_{31} + c_1p_{32} + c_1p_{34} + c_2p'_3 + c_2p'_{31} + c_2p'_{32} + c_2p'_{34} = 1;$

$d_1p_4 + d_1p_{41} + d_1p_{42} + d_1p_{43} + d_2p'_4 = 1.$

Methodology

The mean queue length obtained in ref. (14) is given by:

$$Lq = -\frac{\lambda}{A}\left[\begin{array}{l}\left[1-(c_1p_{34}+c_2p_{34}')d_1p_{43}-(b_1p_{23}+b_2p_{23}')(c_1p_{32})-(b_1p_{23}'+b_2p_{23}')(c_1p_{34}+c_2p_{34}')d_1p_{42}\right] \\ +(a_1p_{12}+a_2p_{12}')[1-(c_1p_{34}+c_2p_{34}')d_1p_{43}]+\dfrac{(a_1p_{12}+a_2p_{12}')(b_1p_{23}+b_2p_{23}')}{[\mu_3-d_1p_{43}\mu_4-(b_1p_{23}+b_2p_{23}')\mu_2]} \\ +\dfrac{(a_1p_{12}+a_2p_{12}')(b_1p_{23}+b_2p_{23}')(c_1p_{34}+c_2p_{34}')}{[\mu_4-\mu_3(c_1p_{34}+c_2p_{34}')]}\end{array}\right]$$

Thus the waiting time of customer for the service in the whole queueing system is:

$$W = -\frac{1}{A}\left[\begin{array}{l}\left[1-(c_1p_{34}+c_2p_{34}')d_1p_{43}-(b_1p_{23}+b_2p_{23}')(c_1p_{32})-(b_1p_{23}'+b_2p_{23}')(c_1p_{34}+c_2p_{34}')d_1p_{42}\right] \\ +(a_1p_{12}+a_2p_{12}')[1-(c_1p_{34}+c_2p_{34}')d_1p_{43}]+\dfrac{(a_1p_{12}+a_2p_{12}')(b_1p_{23}+b_2p_{23}')}{[\mu_3-d_1p_{43}\mu_4-(b_1p_{23}+b_2p_{23}')\mu_2]} \\ +\dfrac{(a_1p_{12}+a_2p_{12}')(b_1p_{23}+b_2p_{23}')(c_1p_{34}+c_2p_{34}')}{[\mu_4-\mu_3(c_1p_{34}+c_2p_{34}')]}\end{array}\right]$$

where $A = [(b_1 p_{21}) (a_1 p_{12}' + a_2 p_{12}') - 1] (c_1 p_{34} + c_2 p_{34}) (d_1 p_{43} + 1) + (b_1 p_{23} + b_2 p_{23}')$

$[c_1 p_{31} - (c_1 p_{34} + c_2 p_{34}') d_1 p_{41} (a_1 p_{12} + a_2 p_{12}') + (c_1 p_{34} + c_2 p_{34}') d_1 p_{42}]$

Numerical Results and Discussion

- Behaviour of waiting time (W) of customer in the system with respect to p_2(the probability of leaving the system first time from second server) for different values of p_3 (probability of leaving the system from third server first time) is depicted in Table 1 keeping the values of other parameters as fixed.

Table 1

$\lambda=2, \mu_1 = 10, \mu_2 = 7, \mu_3= 9, \mu_4=8, a_1=0.3, a_2=0.7, b_1=0.1, b_2=0.9, c_1=0.8, c_2=0.2, d_1=0.6, d_2=0.4, p_1=0.7, p_{12}=0.3, p_1'=0.4, p_{12}'=0.6, p_3=0.3, p_{31}=0.2, p_{32}=0.1, p_{34}=0.4, p_3'=7, p'_{34}=0.3, p_4=0.5, p_{41}=0.1, p_{42}=0.2, p_{43}=0.2, p'_4=1$			
	p_3=0.3	**p_3=0.4**	**p_3=0.45**
p_2	**W**	**W**	**W**
0.3	3.82978093	11.21512736	22.18906615
0.325	3.820670698	11.23123064	22.35192764
0.35	3.811606405	11.24739183	22.51724962
0.375	3.802587754	11.26361126	22.68508799
0.4	3.79361445	11.27988928	22.85550036
0.425	3.784686201	11.29622623	23.02854612
0.45	3.775802719	11.31262244	23.20428648
0.475	3.766963718	11.32907828	23.38278456
0.5	3.758168914	11.34559408	23.56410549
0.525	3.74941803	11.3621702	23.74831643
0.55	3.740710786	11.37880699	23.9354867

Following can be interpreted from **Table 1**:

- Waiting time decreases with the increase in the probability p_2 for the probability p_3**≤**0.3 and increases for p_3>0.3.
- Waiting time increases with the increase in the probability p_3.

- Behaviour of waiting time (W) of customer in the system with respect to p_1(the probability of leaving the system from first server first time) for different values of p'_1 (probability of leaving the system from first server second time) is depicted in Table 2keeping the values of other parameters as fixed.

Table 2

$\lambda=2, \mu_1 = 10, \mu_2 = 7, \mu_3= 9, \mu_4=8, a_1=0.3, a_2=0.7, b_1=0.1, b_2=0.9, c_1=0.8, c_2=0.2, d_1=0.6, d_2=0.4, p_2=0.3, p_{21}=0.2, p_{23}=0.5, p'_2=0.8, p'_{23}=0.2, p_3=0.3, p_{31}=0.2, p_{32}=0.1, p_{34}=0.4, p_3'=7, p'_{34}=0.3, p_4=0.5, p_{41}=0.1, p_{42}=0.2, p_{43}=0.2, p'_4=1$			
	p'_1=0.4	**p'_1=0.6**	**p'_1=0.8**
p_1	**W**	**W**	**W**
0.1	4.280994091	3.90563365	3.531853017
0.15	4.240352722	3.865126594	3.491479505
0.2	4.199721901	3.82463003	3.451116428
0.25	4.159101626	3.784143954	3.410763782
0.3	4.118491892	3.743668362	3.370421564
0.35	4.077892694	3.703203249	3.330089769
0.4	4.037304029	3.662748613	3.289768394
0.45	3.996725893	3.622304448	3.249457434
0.5	3.956158281	3.58187075	3.209156884
0.55	3.915601189	3.541447516	3.168866743
0.6	3.875054614	3.501034742	3.128587004

Following can be interpreted from **Table 2**:

Waiting time decreases with the increase in the probability $p_1$as well as for p'_1.

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