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NON-LINEAR STABILITY OF TRIANGULARLIBRATION POINTS IN THE RESTRICTED THREE BODY PROBLEM WITH VARIABLE MASS AND SMALLER PRIMARY IS A TRIAXIAL RIGID BODY

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ABSTRACT

This paper deals with the non- linear stability of triangular libration points by numerical method. By forming Lagrangian about the triangular librationpoints and taking only second order terms in Lagrangian about triangular libration point the characteristic equations were established by Lagrangian equations of motion. Then by the nature of characteristic roots we found that the triangular libration points are also unstable.

Keywords: *Restricted Three-Body Problem, Jean's law, Space-Time-Transformation, Libration Points, Non-Linear Stability*.

Introduction

Shrivastava and Ishwar (1983)developed the equations of motion of the infinitesimal body of slowly reducing mass with time in the restricted three – body problem with the help of Jeans Law (1928) and Meshcherskii's (1949) space time transformation. Singh et al.(1984) studied the effect of perturbation on the location of libration points in the restricted three – body problem with variable mass and further (1985) he extended his work by discussing the effect of perturbation on the stability of the triangular libration points in the restricted three – body problem with variable mass. Das et al. (1988) established the equation of the infinitesimal body of slowly reducing mass with time in the elliptic restricted three – body problem.

Lukyanov (1990) showed that all types of libration points are stable under the conditions satisfying Meshcherskii's space time transformation. In the same year, El – Shaboury established the equations of motion of the infinitesimal body of reducing mass with time in the gravitational field of the two triaxial primaries with the help of Jeans Law (1928), Meshcherskii's space time transformation (1949) and Nechvili's transformation (1926). Plastino et al. (1992) developed a technique to show the velocity of Newton's second law of motion on the restricted three – body problem with variable mass and found that Newton's second law of motion is invalid for the problem with variable mass.Bekov (1993a, b) discussed the existence and stability of libration points in the R3BP and characterized the Hill's surface. Singh (2008) has also discussed the non – linear stability of libration points in the CR3BP with variable mass and further in 2010 extended his work in 2008 by taking the bigger primary as an oblate spheroid. In both the works, he used the analytical process developed by earlier authors.

In the present work, we have proposed to examine the non-linear stability of triangular libration point by using nature of roots of the characteristic equation corresponding to each triangular libration points.

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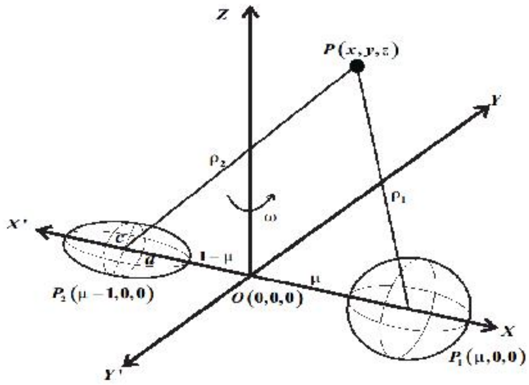

Figure 1: Rotating Frame of Reference in the R3BP in 3-Dimension about Z-axis

Equations of Motion

Following Hassan, Sweta, Thapa and Hassan (2019), the equations of motion of the infinitesimal body in Cartesian form are

$$\left.\begin{aligned} \ddot{x}+\frac{\dot{m}}{m}\left(\dot{x}-\omega y\right)+2\omega\dot{x} &= -\frac{1}{m}\Omega_x ,\\ \ddot{y}+\frac{\dot{m}}{m}\left(\dot{y}+\omega x\right)-2\omega\dot{y} &= -\frac{1}{m}\Omega_y ,\\ \ddot{z}+\frac{\dot{m}}{m}\dot{z} &= -\frac{1}{m}\Omega_z , \end{aligned}\right\} \text{..........} \quad (1)$$

where

$$\Omega=-m\left[\frac{\omega^2}{2}\left(x^2+y^2\right)+\frac{1-\mu}{\rho_1}+\frac{\mu}{\rho_2}-\frac{\mu\left(2\sigma_1-\sigma_2\right)}{2\rho_2^3}-\frac{3\mu\left(2\sigma_1-\sigma_2\right)y^2}{2\rho_2^5}-\frac{3\mu\sigma_1 z^2}{2\rho_2^5}\right]. \text{......} \quad (2)$$

$$\left.\begin{aligned} -\frac{1}{m}\Omega_x = \omega^2 x &-\frac{(1-\mu)(x-\mu)}{\rho_1^3}-\frac{\mu\left(x-\mu+1\right)}{\rho_2^3}-\frac{3\mu\left(2\sigma_1-\sigma_2\right)\left(x-\mu+1\right)}{2\rho_2^5}\\ &+\frac{15\mu\left(\sigma_1-\sigma_2\right)\left(x-\mu-1\right)y^2}{2\rho_2^7}-\frac{15\mu\sigma_1\left(x-\mu-1\right)z^2}{2\rho_2^7}, \end{aligned}\right\} \text{.......} \quad (3)$$

$$\left.\begin{aligned} -\frac{1}{m}\Omega_y &= \omega^2 y-\frac{(1-\mu)y}{\rho_1^3}-\frac{\mu y}{\rho_2^3}-\frac{3\mu\left(2\sigma_1-\sigma_2\right)y}{2\rho_2^5}+\frac{15\mu\left(\sigma_1-\sigma_2\right)y^2}{2\rho_2^7}+\frac{15\mu\sigma_1 yz^2}{2\rho_2^7},\\ \text{and } -\frac{1}{m}\Omega_z &= -\frac{(1-\mu)z}{\rho_1^3}-\frac{\mu z}{\rho_2^3}-\frac{3\mu\left(2\sigma_1-\sigma_2\right)z}{2\rho_2^5}+\frac{15\mu\left(\sigma_1-\sigma_2\right)y^2 z}{2\rho_2^7}+\frac{15\mu\sigma_1 z^3}{2\rho_2^7}. \end{aligned}\right\} \text{......} \quad (4)$$

By Jeans law, the variation of mass of the infinitesimal body is given by

$$\frac{dm}{dt}=-\alpha m^n \quad \text{i.e.,} \quad \frac{\dot{m}}{m}=-\alpha m^{n-1}, \text{..........} \quad (5)$$

where $\sim$ is a constant coefficient and the value of exponent $n \in [0.4,\ \ 4.4]$ for the stars of the main sequence.

Following Hassan&Sweta (2017,2019) the coordinates of triangular libration point $L_4(\xi_4, \eta_4, 0)$ are given by

$$\xi_4 = \mu - \frac{1}{2} + \frac{2 - \frac{7\alpha^2}{3} - \frac{93\sigma_1}{4} + \frac{69\sigma_2}{4}}{\frac{7\alpha^2}{2} + 6 + 69\sigma_1 - 42\sigma_2} + \frac{1}{2}(2\sigma_1 - \sigma_2) \qquad ....(6)$$

$$\eta_4 = +\frac{\sqrt{3}}{2}\left[1 + \frac{2}{3}\left(\frac{2 - \frac{7\alpha^2}{3} - \frac{93\sigma_1}{4} + \frac{69\sigma_2}{4}}{\frac{7\alpha^2}{2} + 6 + 69\sigma_1 - 42\sigma_2}\right) - \frac{1}{2}(2\sigma_1 - \sigma_2)\right]$$

Non – Linear Stability of TriangularLibration Points

Now let us discuss the non – linear stability of the triangularlibration points $L_{4,5}(a, \pm b)$ only, where

$$a = « - \frac{1}{2} + p + q, \quad b = \frac{\sqrt{3}}{2}\left[1 + \frac{2}{3} - p - q\right], \ ....(7)$$

where

$$\left.\begin{aligned} p &= \frac{2 - \frac{7}{3}\sim^2 - \frac{93}{4}\pm_1 + \frac{69}{4}\pm_2}{\frac{7}{2}\sim^2 + \pm + 69\pm_1 - 42\pm_2}, \\ q &= \frac{1}{2}(2\pm_1 - \pm_2), \\ \mu^2 &= 1 + 3q = 1 + \frac{3}{2}(2\pm_1 - \pm_2). \end{aligned}\right\} \quad ....(8) \text{ Hassan and Sweta(2017,2019)}$$

The Lagrangian function of the problem can be written as

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + mw(x\dot{y} - \dot{x}y) + \frac{1}{2}w^2(x^2 + y^2) + m\left[\frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2} - \frac{\mu(2\sigma_1 - \sigma_2)}{2\rho_2^3} - \frac{3\mu(\sigma_1 - \sigma_2)}{2\rho_2^5}y^2\right] ....(9)$$

Now shifting the origin to the libration point $L_4(a,b)$ by taking $x \to x + a$ and $y \to y + b$ then

$$\left.\begin{aligned} \circ_1^2 &= (x + a - «)^2 + (y + b)^2, \\ \circ_2^2 &= (x + a - « + 1)^2 + (y + b)^2, \end{aligned}\right\}$$

and the corresponding Lagrangian about $L_4(a,b)$ is given by

$$\left.\begin{aligned} L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + m\mu[(x+a)\dot{y} - \dot{x}(y+b)] + \frac{1}{2}\mu^2[(x+a)^2 + (y+b)^2] \\ + m\left[\frac{1-«}{\circ_1} + \frac{«}{\circ_2} - \frac{«(2\pm_1 - \pm_2)}{2\circ_2^3} - \frac{3«(\pm_1 - \pm_2)(y+b)^2}{2\circ_2^5}\right]. \end{aligned}\right\} ......(10)$$

Let $f\left(x,y\right)=\frac{1}{\rho_1}=\frac{1}{\left[\left(x+a-\mu\right)^2+\left(y+b\right)^2\right]^{\frac{1}{2}}}$,

$g\left(x,y\right)=\frac{1}{\rho_2}=\frac{1}{\left[\left(x+a-\mu+1\right)^2+\left(y+b\right)^2\right]^{\frac{1}{2}}}$,

$h\left(x,y\right)=\frac{1}{\rho_2^3}$, $\qquad k\left(x,y\right)=\frac{1}{\rho_2^5}$.

$$\rho_1^2=\left(x+a-\mu\right)^2+\left(y+b\right)^2,$$

$$\rho_2^2=\left(x+a-\mu+1\right)^2+\left(y+b\right)^2,$$

$$\frac{\partial\rho_1}{\partial x}=\frac{\left(x+a-\mu\right)}{\rho_1}, \qquad \frac{\partial\rho_2}{\partial x}=\frac{\left(x+a-\mu+1\right)}{\rho_2},$$

$$\frac{\partial\rho_1}{\partial y}=\frac{\left(y+b\right)}{\rho_1}, \qquad \frac{\partial\rho_2}{\partial y}=\frac{\left(y+b\right)}{\rho_2},$$

$$f\left(0,0\right)=\frac{1}{\sqrt{\left(a-\mu\right)^2+b^2}}=\left[\left(a-\mu\right)^2+b^2\right]^{-\frac{1}{2}},$$

$$g\left(0,0\right)=\frac{1}{\sqrt{\left(a-\mu+1\right)^2+b^2}}=\left[\left(a-\mu+1\right)^2+b^2\right]^{-\frac{1}{2}},$$

$$h\left(0,0\right)=\left[\left(a-\mu+1\right)^2+b^2\right]^{-\frac{3}{2}},$$

$$k\left(0,0\right)=\left[\left(a-\mu+1\right)^2+b^2\right]^{-\frac{5}{2}}.$$

$$f_x=-\frac{\left(x+a-\mu\right)}{\rho_1^3},$$

$$f_y=-\frac{\left(y+b\right)}{\rho_1^3},$$

$$f_x\left(0,0\right)=-\frac{\left(a-\mu\right)}{\rho_1^3},$$

$$f_y\left(0,0\right)=-\frac{b}{\rho_1^3},$$

$$f_{xx}=-\frac{1}{\rho_1^3}+\frac{3\left(x+a-\mu\right)^2}{\rho_1^5},$$

$$f_{xy}=\frac{3\left(x+a-\mu\right)\left(y+b\right)}{\rho_1^5},$$

$$f_{yy}=-\frac{1}{\rho_1^3}+\frac{3\left(y+b\right)^2}{\rho_1^5},$$

$$f_{xx}\left(0,0\right)=-\frac{1}{\rho_1^3}+\frac{3\left(a-\mu\right)^2}{\rho_1^5}$$

$$f_{xy}\left(0,0\right)=\frac{3\left(a-\mu\right)b}{\rho_1^3},$$

$$f_{yy}\,(0,0) = -\frac{1}{\rho_1^3} + \frac{3b^2}{\rho_1^5}.$$

$$f_{xxx} = \frac{9(x+a-\mu)}{\rho_1^5} - \frac{15(x+1-\mu)^3}{\rho_1^7},$$

$$f_{xxx}\,(0,0) = -\frac{9(a+\mu)}{\rho_1^5} - \frac{15(a-\mu)^3}{\rho_1^7},$$

$$f_{xxy} = \frac{3(y+b)}{\rho_1^5} - \frac{15(x+a-\mu)^2(y+b)}{\rho_1^7},$$

$$f_{xxy}\,(0,0) = \frac{3b}{\rho_1^5} - \frac{15(a-\mu)^2 b}{\rho_1^7},$$

$$f_{xyy} = \frac{3(x+a-\mu)}{\rho_1^5} - \frac{15(x+a-\mu)(y+b)^2}{\rho_1^7},$$

$$f_{xyy}\,(0,0) = \frac{3(a-\mu)}{\rho_1^5} - \frac{15(a-\mu)b^2}{\rho_1^7},$$

$$f_{yyy} = \frac{9(y+b)}{\rho_1^5} - \frac{15(y+b)^3}{\rho_1^7}.$$

$$f_{yyy}\,(0,0) = \frac{9b}{\rho_1^5} - \frac{15b^3}{\rho_1^7}.$$

$$f_{xxxx} = \frac{9}{\rho_1^5} - \frac{90(x+a-\mu)^2}{\rho_1^7} + \frac{105(x+a-\mu)^4}{\rho_1^9},$$

$$f_{xxxx}\,(0,0) = \frac{9}{\rho_1^5} - \frac{90(a-\mu)^2}{\rho_1^7} + \frac{105(a-\mu)^4}{\rho_1^9},$$

$$f_{xxxy} = -\frac{45(x+a-\mu)(y+b)}{\rho_1^7} + \frac{105(x+a-\mu)^3(y+b)}{\rho_1^9},$$

$$f_{xxxy}\,(0,0) = -\frac{45(a-\mu)(b)}{\rho_1^7} + \frac{105(a-\mu)^3(b)}{\rho_1^9},$$

$$f_{xxyy} = -\frac{12}{\rho_1^5} + \frac{105(x+a-\mu)^2(y+b)^2}{\rho_1^9},$$

$$f_{xxyy}\,(0,0) = -\frac{12}{\rho_1^5} + \frac{105(a-\mu)^2 b^2}{\rho_1^9},$$

$$f_{xyyy} = -\frac{45(x+a-\mu)(y+b)}{\rho_1^7} + \frac{105(x+a-\mu)(y+b)^3}{\rho_1^9},$$

$$f_{xyyy}\,(0,0) = -\frac{45(a-\mu)(b)}{\rho_1^7} + \frac{105(a-\mu)(b)^3}{\rho_1^9},$$

$$f_{yyyy} = \frac{9}{\rho_1^5} - \frac{90(y+b)^2}{\rho_1^7} + \frac{105(y+b)^4}{\rho_1^9},$$

$$f_{yyyy}\,(0,0) = \frac{9}{\rho_1^5} - \frac{90(b)^2}{\rho_1^7} + \frac{105(b)^4}{\rho_1^9}.$$

$$g(x,y)=\frac{1}{\rho_2},$$

$$g_x=-\frac{(x+a-\mu+1)}{\rho_2^3}\Rightarrow g_x(0,0)=-\frac{(a-\mu+1)}{\rho_2^3},$$

$$g_y=-\frac{(y+b)}{\rho_1^3}\Rightarrow g_y(0,0)=-\frac{b}{\rho_2^3},$$

$$g_{xx}=-\frac{1}{\rho_2^3}+\frac{3(x+a-\mu+1)^2}{\rho_2^5},$$

$$g_{xx}(0,0)=-\frac{1}{\rho_2^3}+\frac{3(a-\mu+1)^2}{\rho_2^5},$$

$$g_{xy}=\frac{3(x+a-\mu+1)(y+b)}{\rho_2^5},$$

$$g_{xy}(0,0)=\frac{3(a-\mu+1)(b)}{\rho_2^5},$$

$$g_{yy}=-\frac{1}{\rho_2^3}+\frac{3(y+b)^2}{\rho_2^5},$$

$$g_{yy}(0,0)=-\frac{1}{\rho_2^3}+\frac{3(b)^2}{\rho_2^5},$$

$$g_{xxx}=\frac{9(x+a-\mu+1)}{\rho_2^5}-\frac{15(x+a-\mu+1)^3}{\rho_2^7},$$

$$g_{xxx}(0,0)-\frac{9(a-\mu+1)}{\rho_2^5}-\frac{15(a-\mu+1)^3}{\rho_2^7},$$

$$g_{xxy}=\frac{3(y+b)}{\rho_2^3}-\frac{15(x+a-\mu+1)^2(y+b)}{\rho_2^7}.$$

$$g_{xxy}(0,0)=\frac{3(b)}{\rho_2^5}-\frac{15(a-\mu+1)^2(b)}{\rho_2^7},$$

$$g_{xyy}=\frac{3(a-\mu+1)}{\rho_2^5}-\frac{15(x+a-\mu+1)(y+b)^2}{\rho_2^7}$$

$$g_{xyy}(0,0)=\frac{3(a-\mu+1)}{\rho_2^5}-\frac{15(a-\mu+1)(b)^2}{\rho_2^7},$$

$$g_{yyy}=\frac{9(y+b)}{\rho_2^5}-\frac{15(y+b)^3}{\rho_2^7}.$$

$$g_{yyy}(0,0)=\frac{9(b)}{\rho_2^5}-\frac{15(b)^3}{\rho_2^7}.$$

$$g_{xxxx}=\frac{9}{\rho_2^5}-\frac{90(x+a-\mu+1)^2}{\rho_2^7}+\frac{105(x+a-\mu+1)^4}{\rho_2^9},$$

$$g_{xxxx}(0,0)-\frac{9}{\rho_2^5}-\frac{90(a-\mu+1)^2}{\rho_2^7}+\frac{105(a-\mu+1)^4}{\rho_2^9},$$

$$g_{xxxy}-\frac{45(x+a-\mu+1)(y+b)}{\rho_2^7}+\frac{105(x+a-\mu+1)^3(y+b)}{\rho_2^9},$$

$$g_{xxxy}(0,0)=-\frac{45(a-\mu+1)(b)}{\rho_2^7}+\frac{105(a-\mu+1)^3(b)}{\rho_2^9},$$

$$g_{xxyy} = -\frac{12}{\rho_2^5} + \frac{105(x+a-\mu+1)^2(y+b)^2}{\rho_2^9},$$

$$g_{xxyy}(0,0) = -\frac{12}{\rho_2^5} + \frac{105(a-\mu+1)^2(b)^2}{\rho_2^9},$$

$$g_{xyyy} = -\frac{45(x+a-\mu+1)(y+b)}{\rho_2^7} + \frac{105(x+a-\mu+1)(y+b)^3}{\rho_2^9},$$

$$g_{xyyy}(0,0) = -\frac{45(a-\mu+1)(b)}{\rho_2^7} + \frac{105(a-\mu+1)(b)^3}{\rho_2^9},$$

$$g_{yyyy} = \frac{9}{\rho_2^5} - \frac{90(y+b)^2}{\rho_2^7} + \frac{105(y+b)^4}{\rho_2^9},$$

$$g_{yyyy}(0,0) = \frac{9}{\rho_2^5} - \frac{90(b)^2}{\rho_2^7} + \frac{105(b)^4}{\rho_2^9},$$

$$h(x,y) = \rho_2^{-3},$$

$$h_x = -3\rho_2^{-4}\frac{\partial \rho_2}{\partial x} = -\frac{3(x+a-\mu+1)}{\rho_2^5},$$

$$h_y = -\frac{3(y+b)}{\rho_2^5},$$

$$h_x(0,0) = -\frac{3(a-\mu+1)}{\rho_2^5},$$

$$h_y(0,0) = -\frac{3(b)}{\rho_2^5},$$

$$h_{xx} = -\frac{3}{\rho_2^5} + \frac{15(x+a-\mu+1)^2}{\rho_2^7},$$

$$h_{xx}(0,0) = -\frac{3}{\rho_2^5} + \frac{15(a-\mu+1)^2}{\rho_2^7},$$

$$h_{xy} = \frac{15(x+a-\mu+1)(y+b)}{\rho_2^7},$$

$$h_{xy}(0,0) = \frac{15(a-\mu+1)(b)}{\rho_2^7},$$

$$h_{yy} = -\frac{3}{\rho_2^5} + \frac{15(y+b)^2}{\rho_2^7},$$

$$h_{yy}(0,0) = -\frac{3}{\rho_2^5} + \frac{15(b)^2}{\rho_2^7},$$

$$h(x,y) = \rho_2^{-3},$$

$$h_x = -3\rho_2^{-4}\frac{\partial \rho_2}{\partial x} = -\frac{3(x+a-\mu+1)}{\rho_2^5},$$

$$h_y = -\frac{3(y+b)}{\rho_2^5},$$

$$h_x(0,0) = -\frac{3(a-\mu+1)}{\rho_2^5},$$

$$h_y(0,0) = -\frac{3(b)}{\rho_2^5},$$

$$h_{xx} = -\frac{3}{\rho_2^{5}} + \frac{15(x+a-\mu+1)^2}{\rho_2^{7}},$$

$$h_{xx}(0,0) = -\frac{3}{\rho_2^{5}} + \frac{15(a-\mu+1)^2}{\rho_2^{7}},$$

$$h_{xy} = \frac{15(x+a-\mu+1)(y+b)}{\rho_2^{7}},$$

$$h_{xy}(0,0) = \frac{15(a-\mu+1)(b)}{\rho_2^{7}},$$

$$h_{yy} = -\frac{3}{\rho_2^{5}} + \frac{15(y+b)^2}{\rho_2^{7}},$$

$$h_{yy}(0,0) = -\frac{3}{\rho_2^{5}} + \frac{15(b)^2}{\rho_2^{7}},$$

$$h_{xxx} = \frac{45(x+a-\mu+1)}{\rho_2^{7}} - \frac{105(x+a-\mu+1)^3}{\rho_2^{9}},$$

$$h_{xxx}(0,0) = \frac{45(a-\mu+1)}{\rho_2^{7}} - \frac{105(a-\mu+1)^3}{\rho_2^{9}},$$

$$h_{xxy} = \frac{15(y+b)}{\rho_2^{7}} - \frac{105(x+a-\mu+1)^2(y+b)}{\rho_2^{9}},$$

$$h_{xyy} = \frac{15(x+a-\mu+1)}{\rho_2^{7}} - \frac{105(x+a-\mu+1)(y+b)^2}{\rho_2^{9}},$$

$$h_{xyy}(0,0) = \frac{15(a-\mu+1)}{\rho_2^{7}} - \frac{105(a-\mu+1)(b)^2}{\rho_2^{9}},$$

$$h_{yyy} = \frac{4(y+b)}{\rho_2^{7}} - \frac{105(y+b)^3}{\rho_2^{9}},$$

$$h_{yyy}(0,0) = \frac{4(b)}{\rho_2^{7}} - \frac{105(b)^3}{\rho_2^{9}},$$

$$h_{xxxx} = \frac{45}{\rho_2^{7}} - \frac{630(x+a-\mu+1)^2}{\rho_2^{9}} + \frac{945(x+a-\mu+1)^4}{\rho_2^{11}},$$

$$h_{xxxx}(0,0) = \frac{45}{\rho_2^{7}} - \frac{630(a-\mu+1)^2}{\rho_2^{9}} + \frac{945(a-\mu+1)^4}{\rho_2^{11}},$$

$$h_{xxxy} = -\frac{315(x+a-\mu+1)(y+b)}{\rho_2^{9}} + \frac{945(x+a-\mu+1)^3(y+b)}{\rho_2^{11}},$$

$$h_{xxxy}(0,0) = -\frac{315(a-\mu+1)(b)}{\rho_2^{9}} + \frac{945(a-\mu+1)^3(b)}{\rho_2^{11}},$$

$$h_{xxyy} = -\frac{90}{\rho_2^{7}} + \frac{945(x+a-\mu+1)^2(y+b)^2}{\rho_2^{11}},$$

$$h_{xxyy}(0,0) = -\frac{90}{\rho_2^{7}} + \frac{945(a-\mu+1)^2(b)^2}{\rho_2^{11}},$$

$$h_{xyyy} = -\frac{315(x+a-\mu+1)(y+b)}{\rho_2^{9}} + \frac{945(x+a-\mu+1)(y+b)^3}{\rho_2^{11}},$$

$$h_{xyyy}(0,0) = -\frac{315(a-\mu+1)(b)}{\rho_2^9} + \frac{945(a-\mu+1)(b)^3}{\rho_2^{11}},$$

$$h_{yyyy} = -\frac{45}{\rho_2^7} - \frac{630(y+b)^2}{\rho_2^9} + \frac{945(y+b)^4}{\rho_2^{11}},$$

$$h_{yyyy}(0,0) = -\frac{45}{\rho_2^7} - \frac{630(b)^2}{\rho_2^9} + \frac{945(b)^4}{\rho_2^{11}}.$$

$$k(x,y) = \rho_2^{-5} \Rightarrow k(0,0) = \left[(a-\mu+1)^2 + b^2\right]^{-\frac{5}{2}},$$

$$k_x = -5\rho_2^{-6}\frac{\partial \rho_2}{\partial x} = -\frac{5(x+a-\mu+1)}{\rho_2^7},$$

$$k_y = -\frac{5(y+b)}{\rho_2^7},$$

$$k_x(0,0) = -\frac{5(a-\mu+1)}{\rho_2^7},$$

$$k_y(0,0) = -\frac{5(b)}{\rho_2^7},$$

$$k_{xx} = -\frac{5}{\rho_2^7} + \frac{35(x+a-\mu+1)^2}{\rho_2^9},$$

$$k_{xx}(0,0) = -\frac{5}{\rho_2^7} + \frac{35(a-\mu+1)^2}{\rho_2^9},$$

$$k_{xy} = \frac{35(x+a-\mu+1)(y+b)}{\rho_2^9},$$

$$k_{xy}(0,0) = \frac{35(a-\mu+1)(b)}{\rho_2^9},$$

$$k_{yy} = -\frac{5}{\rho_2^7} + \frac{35(y+b)^2}{\rho_2^9},$$

$$k_{yy}(0,0) = -\frac{5}{\rho_2^7} + \frac{35(b)^2}{\rho_2^9},$$

$$k_{xxx} = \frac{105(x+a-\mu+1)}{\rho_2^9} - \frac{315(x+a-\mu+1)^3}{\rho_2^{11}},$$

$$k_{xxx}(0,0) = \frac{105(a-\mu+1)}{\rho_2^9} - \frac{315(a-\mu+1)^3}{\rho_2^{11}},$$

$$k_{xxy} = \frac{35(y+b)}{\rho_2^9} - \frac{315(x+a-\mu+1)^2(y+b)}{\rho_2^{11}},$$

$$k_{xxy}(0,0) = \frac{35(b)}{\rho_2^9} - \frac{315(a-\mu+1)^2(b)}{\rho_2^{11}},$$

$$k_{xyy} = \frac{35(x+a-\mu+1)}{\rho_2^9} - \frac{315(x+a-\mu+1)(y+b)^2}{\rho_2^{11}},$$

$$k_{xyy}(0,0) = \frac{35(a-\mu+1)}{\rho_2^9} - \frac{315(a-\mu+1)(b)^2}{\rho_2^{11}},$$

$$k_{yyy} = \frac{105(y+b)}{\rho_2^9} - \frac{315(y+b)^2}{\rho_2^{11}},$$

$$k_{yyy}(0,0) = \frac{105(b)}{\rho_2^9} - \frac{315(b)^2}{\rho_2^{11}},$$

$$k_{xxxx}=\frac{105}{\circ^{\frac{9}{2}}}-\frac{1890(x+a-«+1)^2}{\circ^{\frac{11}{2}}}+\frac{3465(x+a-«+1)^4}{\circ^{\frac{13}{2}}},$$

$$k_{xxxx}(0,0)=\frac{105}{\circ^{\frac{9}{2}}}-\frac{1890(a-«+1)^2}{\circ^{\frac{11}{2}}}+\frac{3465(a-«+1)^4}{\circ^{\frac{13}{2}}},$$

$$k_{xxxy}=-\frac{945(x+a-«+1)(y+b)}{\circ^{\frac{11}{2}}}+\frac{3465(x+a-«+1)^3(y+b)}{\circ^{\frac{13}{2}}},$$

$$k_{xxxy}(0,0)=-\frac{945(a-«+1)(b)}{\circ^{\frac{11}{2}}}+\frac{3465(a-«+1)^3(b)}{\circ^{\frac{13}{2}}},$$

$$k_{xxyy}=-\frac{280}{\circ^{\frac{9}{2}}}+\frac{3465(x+a-«+1)^2(y+b)^2}{\circ^{\frac{13}{2}}},$$

$$k_{xxyy}(0,0)=-\frac{280}{\circ^{\frac{9}{2}}}+\frac{3465(a-«+1)^2(b)^2}{\circ^{\frac{13}{2}}},$$

$$k_{xyyy}=-\frac{945(x+a-«+1)(y+b)}{\circ^{\frac{11}{2}}}+\frac{3465(x+a-«+1)(y+b)^3}{\circ^{\frac{13}{2}}},$$

$$k_{xyyy}(0,0)=-\frac{945(a-«+1)(b)}{\circ^{\frac{11}{2}}}+\frac{3465(a-«+1)(b)^3}{\circ^{\frac{13}{2}}},$$

$$k_{yyyy}=\frac{105}{\circ^{\frac{9}{2}}}-\frac{1890(y+b)^2}{\circ^{\frac{11}{2}}}+\frac{3465(y+b)^4}{\circ^{\frac{13}{2}}},$$

$$k_{yyyy}(0,0)=\frac{105}{\circ^{\frac{9}{2}}}-\frac{1890(b)^2}{\circ^{\frac{11}{2}}}+\frac{3465(b)^4}{\circ^{\frac{13}{2}}}.$$

Now putting the values of all the derivatives of $f(x,y), g(x,y), h(x,y)$ and $k(x,y)$ in Equation (10), we get

$$\begin{aligned}
\mathrm{L} = {} & \frac{m}{2}\left(\dot{x}^2+\dot{y}^2\right)+m\,\mu\left(x\dot{y}-\dot{x}y\right)+\mu^2\left(ax+by\right)m+m\,\mu\left(a\dot{y}-b\dot{x}\right)+\frac{\mu^2}{2}\left(x^2+y^2\right)m \\
& +\frac{\mu^2}{2}\left(a^2+b^2\right)m+m\left(1-«\right)\Big[f(0,0)+xf_x(0,0)+yf_y(0,0)+\frac{x^2}{2}f_{xx}(0,0) \\
& +xyf_{xy}(0,0)+\frac{y^2}{2}f_{yy}(0,0)+\frac{x^3}{6}f_{xxx}(0,0)+\frac{x^2y}{2}f_{xxy}(0,0)+\frac{xy^2}{2}f_{xyy}(0,0) \\
& +\frac{y^3}{6}f_{yyy}(0,0)+\frac{x^4}{24}f_{xxxx}(0,0)+\frac{x^3y}{6}f_{xxxy}(0,0)+\frac{x^2y^2}{4}f_{xxyy}(0,0)+\frac{xy^3}{6}f_{xyyy}(0,0) \\
& +\frac{y^4}{24}f_{yyyy}(0,0)\Big]+m\,«\Big[g(0,0)+xg_x(0,0)+yg_y(0,0)+\frac{x^2}{2}g_{xx}(0,0) \\
& +xyg_{xy}(0,0)+\frac{y^2}{2}g_{yy}(0,0)+\frac{x^3}{6}g_{xxx}(0,0)+\frac{x^2y}{2}g_{xxy}(0,0)+\frac{xy^2}{2}g_{xyy}(0,0) \\
& +\frac{y^3}{6}g_{yyy}(0,0)+\frac{x^4}{24}g_{xxxx}(0,0)+\frac{x^3y^6}{6}g_{xxxy}(0,0)+\frac{x^2y^2}{4}g_{xxyy}(0,0) \\
& +\frac{xy^3}{6}g_{xyyy}(0,0)+\frac{y^4}{24}g_{yyyy}(0,0)\Big]-mM\Big[xh_x(0,0)+yh_y(0,0)+\frac{x^2}{2}h_{xx}(0,0) \\
& +h(0,0)+xyh_{xy}(0,0)+\frac{y^2}{2}h_{yy}(0,0)+\frac{x^3}{6}h_{xxx}(0,0)+\frac{x^2y}{2}h_{xxy}(0,0) \\
& +\frac{xy^2}{2}h_{xyy}(0,0)+\frac{y^3}{6}h_{yyy}(0,0)+\frac{x^4}{24}h_{xxxx}(0,0)+\frac{x^3y}{6}h_{xxxy}(0,0)+\frac{x^2y^2}{4}h_{xxyy}(0,0) \\
& +\frac{xy^3}{6}h_{xyyy}(0,0)+\frac{y^4}{24}h_{yyyy}(0,0)\Big]-mN\left(y^2+2by+b^2\right)\Big[k(0,0)++\frac{x^2}{2}k_{xx}(0,0) \\
& +yk_y(0,0)+xk_x(0,0)+xyk_{xy}(0,0)+\frac{y^2}{2}k_{yy}(0,0)+\frac{x^3}{6}k_{xxx}(0,0) \\
& +\frac{x^2y}{2}k_{xxy}(0,0)+\frac{xy^2}{2}k_{xyy}(0,0)+\frac{y^3}{6}k_{yyy}(0,0)+\frac{x^4}{24}k_{xxxx}(0,0)+\frac{x^3y}{6}k_{xxxy}(0,0) \\
& +\frac{x^2y^2}{4}k_{xxyy}(0,0)+\frac{xy^3}{6}k_{xyyy}(0,0)+\frac{y^4}{24}k_{yyyy}(0,0)\Big].
\end{aligned}$$

$$\mathrm{L}=\mathrm{L}_0+\mathrm{L}_1+\mathrm{L}_2+\mathrm{L}_3+\mathrm{L}_4+\mathrm{L}_5+........$$

where

$$L_0 = m(1-«)f(0,0) + m«g(0,0) - mMh(0,0) - Nb^2k(0,0) + \frac{\mu^2}{2}\left(a^2+b^2\right)m,$$

$$L_1 = m\mu(a\dot{y} - b\dot{x}) + \mu^2 m(ax+by) + m(1-«)xf_x(0,0) + m(1-«)yf_y(0,0)$$
$$+ m«xg_x(0,0) + m«yg_y(0,0) - mM\,xh_x(0,0) - mM\,yh_y(0,0) - mNb^2xk_x(0,0)$$
$$- mNb^2yk_y(0,0) - 2mNbyk(0,0),$$

$$L_2 = \frac{m}{2}\left(\dot{x}^2+\dot{y}^2\right) + m\mu(x\dot{y} - \dot{x}y) + \frac{\mu^2}{2}\left(x^2+y^2\right)M + \frac{m(1-«)}{2}x^2f_{xx}(0,0)$$
$$+ m(1-«)xyf_{xy}(0,0) + \frac{m(1-«)}{2}y^2f_{yy}(0,0) + \frac{m«x^2}{2}g_{xx}(0,0) + m«xyg_{xy}(0,0)$$
$$+ \frac{m«y^2}{2}g_{yy}(0,0) - mM\frac{x^2}{2}h_{xx}(0,0) - mMxyh_{xy}(0,0) - mM\frac{y^2}{2}h_{yy}(0,0)$$
$$- mNy^2k(0,0) - 2Nbxyk_x(0,0) - 2Nby^2k_y(0,0) - mNb^2\frac{x^2}{2}k_{xx}(0,0)$$
$$- mNb^2xyk_{xy}(0,0) - \frac{mNb^2y^2}{2}k_{yy}(0,0),$$

$$L_3 = \frac{m(1-«)}{6}x^3f_{xxx}(0,0) + \frac{m(1-«)}{2}x^2yf_{xxy}(0,0) + \frac{m(1-«)}{2}xy^2f_{xyy}(0,0)$$
$$+ \frac{m(1-«)}{6}y^3f_{yyy}(0,0) + \frac{m«}{6}x^3g_{xxx}(0,0) + \frac{m«}{2}yg_{xxy}(0,0) + \frac{m«}{2}xy^2g_{xyy}(0,0)$$
$$+ \frac{m«}{6}y^3g_{yyy}(0,0) - \frac{mM}{6}x^3h_{xxx}(0,0) - \frac{mM}{2}x^2yh_{xxy}(0,0) - \frac{mMxy^2}{2}h_{xyy}(0,0)$$
$$- \frac{mMy^3}{6}h_{yyy}(0,0) - mNxy^2(0,0) - mNy^3k_y(0,0) - mNbx^2yk_{xx}(0,0)$$
$$- 2mNbxy^2k_{xy}(0,0) - mNby^3k_{yy}(0,0) - \frac{mNb^2x^3}{6}k_{xxx}(0,0) - \frac{mN}{2}b^2x^2yk_{xxy}(0,0)$$
$$- \frac{mN}{2}b^2xy^2k_{xyy}(0,0) - \frac{mN}{6}b^2y^3k_{yyy}(0,0),$$

$$L_4 = \frac{m(1-«)}{24}x^4f_{xxxx}(0,0) + \frac{m(1-«)}{6}x^3yf_{xxxy}(0,0) + \frac{m(1-«)}{4}x^2y^2f_{xxyy}(0,0)$$
$$+ \frac{m(1-«)}{6}xy^3f_{xyyy}(0,0) + \frac{m(1-«)}{24}y^4f_{yyyy}(0,0) + \frac{m«}{24}x^4g_{xxxx}(0,0)$$
$$+ \frac{m«}{6}x^3yg_{xxxy}(0,0) + \frac{m«}{4}x^2y^2g_{xxyy}(0,0) + \frac{m«}{6}xy^3g_{xyyy}(0,0) + \frac{m«}{4}y^4g_{yyyy}(0,0)$$
$$- \frac{mM}{24}x^4h_{xxxx}(0,0) - -\frac{mM}{6}x^3yh_{xxxy}(0,0) - -\frac{mM}{4}x^2y^2h_{xxyy}(0,0)$$
$$- \frac{mM}{6}xy^3h_{xyyy}(0,0) - \frac{mM}{24}y^4h_{yyyy}(0,0) - \frac{mNb^2}{24}x^4k_{xxxx}(0,0)$$
$$- \frac{mNb^2}{6}x^3yk_{xxxy}(0,0) - \frac{mNb^2}{4}x^2y^2k_{xxyy}(0,0) - \frac{mNb^2}{6}xy^3k_{xyyy}(0,0)$$
$$- \frac{mNb^2}{24}y^4k_{yyyy}(0,0) - \frac{mNb^2}{24}x^4k_{xxxx}(0,0) - \frac{mNb^2}{6}x^3yk_{xxxy}(0,0)$$
$$- \frac{mNb^2}{4}x^2y^2k_{xxxx}(0,0) - \frac{mNb^2}{6}xy^3k_{xyyy}(0,0) - \frac{mNb^2}{24}y^4k_{yyyy}(0,0)$$
$$- \frac{mN}{2}x^2y^2k_{xx}(0,0) - mNxy^3k_{xy}(0,0) - \frac{mN}{2}y^4k_{yy}(0,0) - \frac{mNb}{3}x^3yk_{xxx}(0,0)$$
$$- mNbx^2y^2k_{xxy}(0,0) - mNbxy^3k_{xyy}(0,0) - \frac{mNb}{3}y^4k_{yyy}(0,0),$$

$L_5 =$ ....,

$L_6 =$ ....

where $M = \frac{«}{2}\left(2\pm_1 - \pm_2\right),\ N = \frac{3«}{2}\left(\pm_1 - \pm_2\right)$.

Now considering L_2 as the Lagrangian function for checking non-linear stability of the libration points,

$$L_2 = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) + m\mu\left(x\dot{y} - \dot{x}y\right) + m\left[\frac{\mu^2}{2} + \frac{1-«}{2}f_{xx}(0,0) + \frac{«}{2}g_{xx}(0,0) - \frac{M}{2}h_{xx}(0,0)\right.$$
$$\left. - \frac{Nb^2}{2}k_{xx}(0,0)\right]x^2 + m\left[(1-«)f_{xy}(0,0) + «g_{xy}(0,0) - Mh_{xy}(0,0)\right.$$
$$\left. - 2Nbk_x(0,0)\right]xy + m\left[\frac{\mu^2}{2} + \frac{1-«}{2}f_{yy}(0,0) + \frac{«}{2}g_{yy}(0,0) - \frac{M}{2}h_{yy}(0,0)\right.$$
$$\left. - \frac{Nb^2}{2}k_{yy}(0,0) - 2Nbk_y(0,0) - Nk(0,0)\right]y^2.$$

$$\left.\begin{aligned}\frac{\partial L_2}{\partial \dot{x}} &= m\left(\dot{x} - \mu y\right),\\ \frac{\partial L_2}{\partial \dot{y}} &= m\left(\dot{y} + \mu x\right).\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(11)$$

$$\frac{\partial L_2}{\partial x} = m\mu\dot{y} + m\left[\mu^2 + (1-«)f_{xx}(0,0) + «g_{xx}(0,0) - Mh_{xx}(0,0)\right.$$
$$\left. - Nb^2k_{xx}(0,0)\right]x + m\left[(1-«)f_{xy}(0,0) + «g_{xy}(0,0) - Mh_{xy}(0,0)\right.$$
$$\left. - 2Nbk_x(0,0) - Nb^2k_{xy}(0,0)\right]y,$$
$$\frac{\partial L_2}{\partial y} = m\left[(1-«)f_{xy}(0,0) + «g_{xy}(0,0) - Mh_{xy}(0,0) - 2Nbk_x(0,0)\right.$$
$$\left. - Nb^2k_{xx}(0,0)\right]x - m\mu\dot{x} + m\left[\mu^2 + (1-«)f_{yy}(0,0) + «g_{yy}(0,0)\right.$$
$$\left. - Mh_{yy}(0,0) - Nb^2k_{yy}(0,0) - 4Nbk_y(0,0) - 2Nk(0,0)\right]y. \quad \ldots\ldots(12)$$

The Lagrange's equations of motion of the first order are

$$\left.\begin{aligned}\frac{d}{dt}\left(\frac{\partial L_2}{\partial \dot{x}}\right) - \frac{\partial L_2}{\partial x} &= 0,\\ \frac{d}{dt}\left(\frac{\partial L_2}{\partial \dot{y}}\right) - \frac{\partial L_2}{\partial y} &= 0.\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(13)$$

$$\left.\begin{aligned}\ddot{x} + \sim\dot{x} - 2\mu\dot{y} &= P_x + (Q + \sim\mu)y,\\ \ddot{y} + 2\mu\dot{x} + \sim\dot{y} &= (Q - \sim\mu)x + Z_y.\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(14)$$

where

$$\left.\begin{aligned}P &= \mu^2 + (1-«)f_{xx}(0,0) + «g_{xx}(0,0) - Mh_{xx}(0,0) - Nb^2k_{yy}(0,0),\\ Q &= (1-«)f_{xy}(0,0) + «g_{xy}(0,0) - Mh_{xy}(0,0) - Nb^2k_{xy}(0,0) - 2Nk_x(0,0),\\ Z &= \mu^2 + (1-«)f_{yy}(0,0) + «g_{yy}(0,0) - Mh_{yy}(0,0) - Nb^2k_{yy}(0,0)\\ &\quad - 4Nbk_y(0,0) - 2Nk(0,0).\end{aligned}\right\} \ldots\ldots(15)$$

Let $x = Ae^{ªt}$ and $y = Be^{ªt}$ be the two solutions of the system (15) then

$$Aª^2e^{ªt} + \sim Aªe^{ªt} - 2\mu Bªe^{ªt} = PAe^{ªt} + (Q + \mu\sim)Be^{ªt},$$
$$Bª^2e^{ªt} + 2\mu Aªe^{ªt} + \sim Bªe^{ªt} = (Q - \sim\mu)Ae^{ªt} + ZBe^{ªt}.$$

$$\left.\begin{aligned}Aª^2 + \sim Aª - 2\mu ªB &= PA + (Q + \mu\sim)B,\\ Bª^2 + 2\mu Aª + \sim Bª &= (Q - \sim\mu)A + ZB.\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Eliminating A and B from the above equations (16), we get the characteristic equation in biquadratic form as

$$a^4 + 2\sim a^3 - \left(4\mu^2 + P + Z - \sim^2\right) a^2 + \sim\left(4\mu^2 - P\right) + \left(\mu^2 \sim^2 - Q^2\right) = 0, \text{ ....(17)}$$

where $\sim = \frac{\dot{m}}{m}, P, Q, Z$ are given in the system (15).

$$M = \frac{«}{2}\left(2\pm_1 - \pm_2\right), \qquad N = \frac{3«}{2}\left(\pm_1 - \pm_2\right),$$

$$f_{xx}(0,0) = -\frac{(a-«)}{\circ_1^3}, \qquad g_{xx}(0,0) = -\frac{(a-«+1)}{\circ_2^3},$$

$$h_{xx}(0,0) = -\frac{3(a-«+1)}{\circ_2^5}, \qquad k_{xx}(0,0) = -\frac{5(a-«+1)}{\circ_2^7},$$

$$f_{xy}(0,0) = \frac{3(a-«)b}{\circ_1^5}, \qquad g_{xy}(0,0) = \frac{3(a-«+1)b}{\circ_2^5},$$

$$h_{xy}(0,0) = \frac{15(a-«+1)b}{\circ_2^7}, \quad k_{xy}(0,0) = \frac{35(a-«+1)b}{\circ_2^9}$$

$$k_x(0,0) = -\frac{5(a-«+1)}{\circ_2^7},$$

$$f_{yy}(0,0) = -\frac{1}{\circ_1^3} + \frac{3b^2}{\circ_1^5}, \qquad g_{yy}(0,0) = -\frac{1}{\circ_2^3} + \frac{3b^2}{\circ_2^5},$$

$$h_{yy}(0,0) = -\frac{3}{\circ_2^5} + \frac{15b^2}{\circ_2^7}, \qquad k_{yy}(0,0) = -\frac{5}{\circ_2^5} + \frac{35b^2}{\circ_2^9},$$

$$k_y(0,0) = -\frac{5b}{\circ_2^7}, \qquad k(0,0) = \left[(a-«+1)^2 + b^2\right]^{-\frac{5}{2}}.$$

$$\circ_1 = \left[(a-«)^2 + b^2\right]^{-\frac{1}{2}}, \qquad \circ_2 = \left[(a-«+1)^2 + b^2\right]^{-\frac{1}{2}}.$$

$$a = \mu - \frac{1}{2} + p + q,$$

$$b = \frac{\sqrt{3}}{2}\left[1 + \frac{2}{3}p - q\right],$$

$$p = \frac{2 - \frac{7}{3}\alpha^2 - \frac{93}{4}\sigma_1 + \frac{69}{4}\sigma_2}{\frac{7}{2}\alpha^2 + 6 + 69\sigma_1 - 42\sigma_2},$$

$$q = \frac{1}{2}(2\sigma_1 - \sigma_2),$$

$$\sigma_1 = \frac{a^2 - c^2}{5R^2},$$

$$\sigma_2 = \frac{b^2 - c^2}{5R^2},$$

a, b, c are the semi-axes of the triaxial rigid bodies. R is the dimensional distance between the centre of two primaries.

Table 1: Characteristics roots and the nature of

Non – Linear Stability of tringular Libration Point $L_4(a,b,0)$ at $I = 0.01,\ « = 0.019$.

Sl. No.	~	Corresponding Characteristics roots (a_1, a_2, a_3, a_4)	Nature
01	0.3	-2.73565, -0.404032, 0.422239, 2.11744	Unstable
02	0.2	-2.64404, -0.324772, 0.332205, 2.23661	Unstable
03	0.02	-2.48073, -0.0985959, 0.0986603, 2.44066	Unstable
04	0.1	-2.55748, -0.22576, 0.227479, 2.33576	Unstable
05	0.01	-2.47172, -0.068752, 0.0687676, 2.4517	Unstable
06	0.001	-2.46362, -0.0162597, 0.0162598, 2.46162	Unstable
07	0.0001	-2.46281, -2.96966*10^{-9}-0.0134206i, 2.9696*10^{-9}+0.0134206i, 2.4262061	Unstable
08	0.00001	-2.46273, -3.70217*10^{-10}-0.0149848i, 3.70217*10^{-10}+0.0149848i, 2.42273	Unstable
09	0.000001	-2.46272, -3.78274*10^{-12}-0.015147i, 3.78274*10^{-12}+0.015147i, 2.42272	Unstable
10	0.0000001	-2.46272, -3.78274*10^{-12}-0.015147i, 3.78274*10^{-12}+0.015147i, 2.42272	Unstable

Discussions and Conclusion

In non – linear stability, it is to be noted that there is a difference of conditions of linear and non – linear stability of the libration points. While we examined the linear stability of libration points three parameters ¥, ~, I were present in the characteristic equations. By keeping ~, $±_1$, $±_2$ fixed and ¥ varies, it is found that all the collinear libration points are unstable in linear sense but triangular libration points are stable for few values of ¥ and stable for some values of ¥ in linear sense.

Here, we should also produce a data for linear stability by keeping ¥ & I fixed and ~ varies within its range. In case of non – linear stability, only three parameters ~, $±_1$, $±_2$ were present in the corresponding characteristic equation. Now by taking I as constant for any particular primary and ~ allowed to vary within its range, it is found that collinear and triangular libration points are unstable.

In some cases, the triangular libration points may be stable with the variation of ~ and I both. For a particular primary, I is fixed but ~ varies for all primaries because ~ is an infinitesimal based parameter whereas I is a primary based parameter. Thus, we conclude that for a few values of ~ , the triangular libration points are stable in non – linear sense.

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