

Dynamic Manpower Planning Models: A Mathematical Framework for Sustainable Organizational Growth

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ABSTRACT

This paper presents a comprehensive mathematical framework for dynamic manpower planning aimed at supporting sustainable organizational growth. The proposed model integrates workforce inflows, internal transitions, skill development, attrition, and demand variability within a multi-period planning horizon. Using systems of difference equations and optimization techniques, the framework enables organizations to forecast workforce requirements, evaluate policy alternatives, and balance cost efficiency with long-term sustainability. The model accounts for both quantitative and qualitative aspects of manpower, allowing decision-makers to align human resource strategies with organizational objectives while adapting to external and internal changes. The applicability of the framework is demonstrated through theoretical analysis, highlighting its potential to improve workforce stability, productivity, and resilience. This research contributes to the existing manpower planning literature by offering a flexible, analytically robust approach suitable for modern, growth-oriented organizations.

Keywords: Dynamic manpower planning, Mathematical modeling, Workforce optimization, Sustainable growth, Human resource planning.

Introduction

Dynamic manpower planning focuses on the systematic analysis of workforce flows—such as recruitment, promotion, training, attrition, and retirement—over multiple time periods. Unlike static models that assume fixed demand and supply conditions, dynamic models explicitly incorporate time-dependent variables and feedback mechanisms. This allows organizations to evaluate how present workforce decisions influence future capabilities, costs, and performance. Mathematical modeling plays a central role in this process by translating organizational realities into quantifiable structures, enabling managers to simulate alternative scenarios and assess trade-offs between competing objectives.

Sustainable organizational growth depends not only on short-term efficiency but also on the long-term availability of skills, leadership pipelines, and institutional knowledge. Poor manpower planning can lead to skill shortages, excessive labor costs, employee burnout, or underutilization of talent. Dynamic models help mitigate these risks by capturing the interdependencies between workforce size, skill composition, productivity, and learning effects. Through tools such as difference equations, stochastic processes, system dynamics, and optimization techniques, organizations can design manpower policies that balance growth ambitions with workforce sustainability.

A mathematical framework for dynamic manpower planning also supports evidence-based human resource management. By formalizing assumptions and constraints, mathematical models reduce reliance on intuition alone and promote consistency in planning decisions. They allow for the integration of uncertainty—such as fluctuating demand, varying attrition rates, or policy changes—into the planning process. Moreover, these models facilitate sensitivity analysis, helping decision-makers understand which

parameters have the greatest impact on workforce outcomes and where managerial intervention is most effective.

As organizations increasingly pursue sustainability as a strategic priority, manpower planning models must account for social and economic dimensions alongside operational performance. Dynamic mathematical frameworks enable the evaluation of long-term workforce stability, equitable career progression, and continuous skill development, all of which are critical for resilient growth. This introduction sets the foundation for exploring dynamic manpower planning models, emphasizing their mathematical structure, practical relevance, and role in supporting sustainable organizational development in an uncertain and competitive landscape.

Development of Manpower Modeling & Optimization

An industry's capacity to predict its workforce needs is essential. On one hand, these predictions help businesses with their workforce planning methods, while on the other, they provide job searchers a way to gauge the desirability of a certain industry. Government policymakers may benefit greatly from labor demand and supply forecasts because they help them avoid duplicative investments and promote sustainable, balanced growth in their respective industries. Under- or over-supply of labor, for example, might result from predictions based on faulty market research. The majority of the time, inaccurate forecasts of labor supply and demand is caused by static and unilateral analysis. When it comes to cutting-edge and emerging industries, these methods can produce incredibly inaccurate predictions. These sectors are known for their complex interdependencies with other industries, rapidly increasing manpower demand, and the length of time it takes to develop the necessary human resources. Making decisions in every sector requires guesswork on demand and performance in the future. For instance, factors such as the amount to manufacture, the amount of capacity and other resources to acquire, the goods to create, the amount of funding required by the firm, and so on. On occasion, decision-makers have a tendency to make the simplistic prediction, thinking that the future would follow the same pattern as the past. The use of mathematical models or statistical methods to predict future demand and supply is also common in other contexts. Unfortunately, dynamic structural analysis and the detection of delayed feedback effects are not yet possible with the current approaches because of their inherent shortcomings.

Creating “Mathematical Modeling

We have by now understood the role of a “Mathematical Model” in addressing challenges in an organization. Now let us understand the process involved in creating a “Mathematical Model”.

Step 1: Collect organization data of projects with respect to the X's as defined in sec 3.6. Note that not all the X's defined in sec 3.6 will be measured in the projects and thus only project level data for measured X's would be available. Also, by way of process consultancy to projects it has to be enforced that certain critical X's for the project must be measured.

Step 2: Sanitize the data to eliminate any data format errors in the data dump. The typical activities here involve checking for blanks in the data, the cell formats are not consistent therefore numbers get treated as text, data time entries are not accurate or missing. Also there are cases where some data from table is left shifted or right shifted. These have to be manually checked most of the time. Larger corporations with the means and resources do use data bases with queries capable of sanitizing the data in the database before it is extracted.

Step 3: Conduct “statistical tests” on the data to identify its mean, “Standard Deviation”, control limits and range. The outliers values have to be removed from the data set as they are not the true situation but an anomaly which requires different treatment and its “Root Cause Analysis”. Also these outliers are generally a one off case and do not represent the actual process performance. We also need to check the data for stability and normality using mathematical tools like Anderson – Darling tests. Depending on whether data is normal or not the “statistical treatments” may differ. However, we can safely assume for non-normal data that after significant data is collected it will move towards normality. This can be corroborated by the Central Limits Theorem.

Step 4: Using the data captured compute metrics which are critical for the project as defined and which will directly contribute to the organizations “business objectives” and the challenge which the organization is trying to address using the “Mathematical Model”. These metrics are the X's as defined in the “Tier I” model for “Profitable Growth” or “Customer Satisfaction”. This way the oscillating / roll-up of the “Mathematical Models” between the tiers can be established.

Step 5: Using Liner Regression prepare a “Mathematical Model”.

Step 6: Deploy the prediction model in the projects of “Tier III” to address the challenge in

question. It may be required to calibrate the "Mathematical Model" to suit the project.

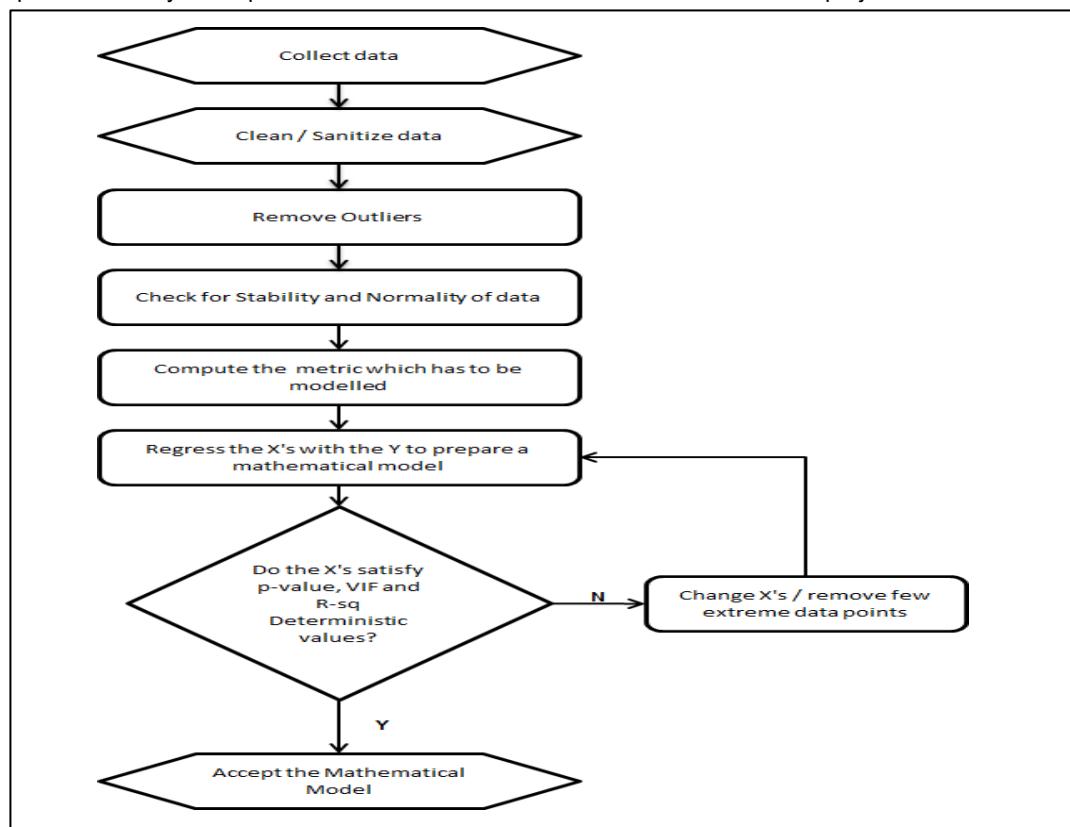


Fig 1: Flowchart for "Mathematical Modelling"

Steel structure manufacturing companies have a significant responsibility in production planning and scheduling, and they rely on component processing times as a reference when they construct production plans overall and decide which nodes to use for certain components. Production planning and complete process scheduling are two areas where businesses rely heavily on accurate man-hour predictions. What this means is that the processing time is unaltered when the same kind of machine is used to process the same components and operations. As a result, production hours fluctuate uncontrollably, leading to a certain deviation between the actual processing hours and the used processing hours.

DETAILS OF THE MODEL

It is thought of as an organization with two tiers that makes policy choices at random intervals. Some people leave the company at random intervals between decision-making epochs. A person's resignation results in a loss of man-hours for the company. People are free to move between grades as they choose. The distribution of thresholds follows an exponential distribution, and each grade has its own unique level. Neither of their different criteria affects the loss of man-hours. An exponential distribution is thought to govern the loss of man-hours. The intervals between decisions are exponentially correlated and swappable variables when the total number of lost man-hours above the sum of two grade criteria, recruiting is initiated.

Notations

T_i : the interval between choices $i = 1, 2, \dots$, with $f(\cdot)$ and $F(\cdot)$ as its cumulative distribution function and probability density function, respectively

EXPECTED TIME AND VARIANCE OF TIME TO RECRUITMENT

Both the variance and the anticipated time to recruitment may be expressed analytically. When the total number of lost man-hours is more than the sum of Y_1 and Y_2 , recruiting is initiated. Assuming

that time t has passed, the likelihood that the human resources system would fail is given by

$$P(W,t) = \sum_{k=0}^{\infty} k P(\sum_{i=1}^k (X_i 1 + X_i 2) < (Y_1 + Y_2))$$

{The likelihood that inside the interval $(0, t]$ there are precisely k policy decisions} expectation that the total harm to the workforce system will not surpass a certain point.

$$= \sum_{k=0}^{\infty} [F_k(t) - F_k + 1(t)] P[\sum_{i=1}^{\infty} (X_i 1 + X_i 2) < (Y_1 + Y_2)]$$

Given that Y_1 and Y_2 are parameters θ_1 and θ_2 in an exponential distribution, the probability density function of $Y_1 + Y_2$ is

$$\begin{aligned} &= \int_0^y k 1(u) k 2(y-u) du \\ &= \int_0^y \theta_1 e^{-\theta_1 u} \theta_2 e^{-\theta_2(y-u)} du \\ &= \frac{\theta_1 \theta_2}{(\theta_1 - \theta_2)} (e^{-\theta_2 y} e^{-\theta_1 y}) \end{aligned}$$

Now,

$$\begin{aligned} P[\sum_{i=1}^k (X_i 1 + X_i 2) < (Y_1 + Y_2)] &= \frac{\theta_1 \theta_2}{(\theta_1 - \theta_2)} \int_0^{\infty} (e^{-\theta_2 y} e^{-\theta_1 y}) Q_k(y) dy \\ &= \frac{\theta_1 \theta_2}{(\theta_1 - \theta_2)} \int_0^{\infty} e^{-\theta_2 y} Q_k(y) dy - \int_0^{\infty} e^{-\theta_1 y} Q_k(y) dy \\ &= \frac{\theta_1 [q*(\theta_2) - \theta_2 [q*(\theta_1)] k]}{(\theta_1 - \theta_2)} \end{aligned}$$

One possible form for the equation is as

$$\begin{aligned} P(W > t) &= \sum_{k=0}^{\infty} [F_k(t) - F_k + 1(t)] \left[\frac{\theta_1 [q*(\theta_2) - \theta_2 [q*(\theta_1)] k]}{(\theta_1 - \theta_2)} \right] \\ &= 1 - \left(\frac{\theta_1}{(\theta_1 - \theta_2)} \right) \left\{ \sum_{k=1}^{\infty} \left[F_k(t) [q * (\theta_2)]^{k-1} - \sum_{k=1}^{\infty} [F_k(t) [q * (\theta_2)]^k] \right] \right\} \\ &\quad + \left(\frac{\theta_1}{(\theta_1 - \theta_2)} \right) \left\{ \sum_{k=1}^{\infty} \left[F_k(t) [q * (\theta_1)]^{k-1} - \sum_{k=1}^{\infty} [F_k(t) [q * (\theta_1)]^k] \right] \right\} \\ &= 1 - \left(\frac{\theta_1}{(\theta_1 - \theta_2)} \right) [1 - q * (\theta_2)] \sum_{k=1}^{\infty} F_k(t) [q * (\theta_2)]^{k-1} \\ &\quad + \left(\frac{\theta_1}{(\theta_1 - \theta_2)} \right) [1 - q * (\theta_1)] \sum_{k=1}^{\infty} F_k(t) [q * (\theta_1)]^{k-1} \\ L(t) &= 1 - P(W > t) \\ &= \left(\frac{\theta_1}{(\theta_1 - \theta_2)} \right) [1 - q * (\theta_2)] \sum_{k=1}^{\infty} F_k(t) [q * (\theta_2)]^{k-1} \\ &\quad + \left(\frac{\theta_1}{(\theta_1 - \theta_2)} \right) [1 - q * (\theta_1)] \sum_{k=1}^{\infty} F_k(t) [q * (\theta_1)]^{k-1} \end{aligned}$$

We get by applying the Laplace-Stieltjes transform on both sides.

$$\begin{aligned} L^*(s) &= \left(\frac{\theta_1}{(\theta_1 - \theta_2)} \right) [1 - q * (\theta_2)] \sum_{k=1}^{\infty} F_k(s) [q * (\theta_2)]^{k-1} \\ &\quad - \left(\frac{\theta_2}{(\theta_1 - \theta_2)} \right) [1 - q * (\theta_1)] \sum_{k=1}^{\infty} F_k(s) [q * (\theta_1)]^{k-1} \end{aligned}$$

Does $\{T_i\}$ follow the exponential distribution with a probability density function for each of its exchangeable and constantly connected random variables?

$$f(t) = \left(\frac{1}{a} \right) e^{-\frac{t}{a}}, a > 0, 0 < t < \infty$$

Based on Gurland's (1995) finding, we may determine the characteristic function of $\{T_i\}$ as

$$\Psi(Y_1, Y_2, \dots, Y_k) = \begin{vmatrix} 1 - iY_1 a & -iY_1 a R & -iY_1 a R \\ -Y_2 a R & 1 - iY_2 a & -iY_2 a R \\ -iY_k a R & -iY_k a R & 1 - iY_k a \end{vmatrix}^{-1}$$

in addition to the cumulative distribution parameter of

$$\sum_{i=1}^k T_i$$

As

$$\begin{aligned}
 Fk(t) &= \frac{1-R}{1-R+kR} \sum_{i=0}^{\infty} \left(\frac{kR}{1-R+kR}\right)^i \frac{\Psi(k+i, t/c)}{(k+i-1)!} \\
 &= \frac{1-R}{1-R+kR} \sum_{i=0}^{\infty} \left(\frac{kR}{1-R+kR}\right)^i \frac{\Psi(k+i, t/c)}{(k+i-1)!} \int_0^{t/c} e^{-x} x^{k+i+1} dx \\
 &= \frac{1-R}{1-R+kR} \sum_{i=0}^{\infty} \left(\frac{kR}{1-R+kR}\right)^i [1 - \sum_{j=0}^{k+i+1} \left(\frac{e^{-t/c}(t/c)^{k+i+1}}{(k+i-j-1)!}\right)]
 \end{aligned}$$

With the Laplace-Stieltjes transform applied to both sides, we get

$$\begin{aligned}
 F^*k(s) &= \frac{s(1-R)}{1-R+kR} \sum_{i=0}^{\infty} \left(\frac{kR}{1-R+kR}\right)^i \\
 &\quad \int_0^{t/c} e^{-st} [1 - \sum_{j=0}^{k+i+1} \left(\frac{e^{-t/c}(t/c)^{k+i+1}}{(k+i-j-1)!}\right)] dt \\
 &= \frac{s(1-R)}{1-R+kR} \sum_{i=0}^{\infty} \left(\frac{kR}{1-R+kR}\right)^i \left[\frac{1}{s} - \frac{c}{1+cs} \sum_{j=0}^{k+i-1} (1+cs)^{-j} \right] \\
 &= \frac{(1-R)}{1-R+kR} \sum_{i=0}^{\infty} \left(\frac{kR}{1-R+kR}\right)^i (1+cs)^{-(k+i)} \\
 &= \frac{(1-R)}{1-R+kR} \sum_{i=0}^{\infty} \frac{1}{(1+cs)^k} \left[\frac{kR}{(1-R+kR)1+cs} \right]^i \\
 &= \frac{(1-R)(1+cs)^{1-k}}{(1-R)(1+cs) + kRcs} \\
 [d/ds (F^*k(s))]_{s=0} &= \frac{c(1-k)(1-R)^2 - c(1-R)^2 - kRc(1-R)}{(1-R)^2} \\
 &= -kc/(1-R) \\
 &= -ak
 \end{aligned}$$

$$\begin{aligned}
 [d^2/ds^2 (F^*k(s))]_{s=0} &= c^2 \left\{ 2k + k(k-1) + \frac{2kR}{1-R} + \frac{2k^2R}{1-R} + \frac{2k^2R^2}{(1-R)^2} \right\} \\
 &= c^2 \left\{ k^2 + k + \frac{2kR}{1-R} + \frac{2k^2R}{1-R} + \left(1 + \frac{R}{1-R}\right) \right\} \\
 &= c^2 \left\{ k^2 \left(1 + \frac{R}{(1-R)^2}\right) + k \left(1 + \frac{2R}{1-R}\right) \right\} \\
 &= c^2 \left\{ k^2 \left(1 + \frac{R^2}{(1-R)^2}\right) + k \left(\frac{1+R}{1-R}\right) \right\}
 \end{aligned}$$

By plugging in the values, we get

$$\begin{aligned}
 E(W) &= -[d/ds (L^*(s))]_{s=0} \\
 &= \left(\frac{\theta_2}{(\theta_1-\theta_2)} \right) [1 - q * (\theta_2)] \sum_{k=1}^{\infty} ak(s) [q * (\theta_2)]^{k-1} \\
 &\quad - \left(\frac{\theta_2}{(\theta_1-\theta_2)} \right) [1 - q * (\theta_1)] \sum_{k=1}^{\infty} ak(s) [q * (\theta_1)]^{k-1} \\
 &= \left(\frac{a}{(\theta_1-\theta_2)} \right) \left\{ \frac{\theta_1}{1-q*(\theta_2)} - \frac{\theta_2}{1-q*(\theta_1)} \right\} \\
 E(W^2) &= -[d^2/ds^2 (L^*(s))]_{s=0} \\
 &= \left(\frac{\theta_2}{(\theta_1-\theta_2)} \right) [1 - q * (\theta_2)] \sum_{k=1}^{\infty} c^2 [q * (\theta_2)]^{k-1} \left(\frac{k^2(1+R^2)}{(1-R)^2} + \frac{k(1+R)}{1-R} \right) \\
 &\quad - \left(\frac{\theta_2}{(\theta_1-\theta_2)} \right) [1 - q * (\theta_1)] \sum_{k=1}^{\infty} c^2 [q * (\theta_1)]^{k-1} \left(\frac{k^2(1+R^2)}{(1-R)^2} + \frac{k(1+R)}{1-R} \right) \\
 &= \left(\frac{c^2\theta_1}{\theta_1-\theta_2} \right) \left(\frac{1+R^2}{(1-R)^2} \right) [1 - q * (\theta_2)] \sum_{k=1}^{\infty} k^2 [q * (\theta_2)]^{k-1} \\
 &\quad + \left(\frac{c^2\theta_1}{\theta_1-\theta_2} \right) \left(\frac{1+R}{1-R} \right) [1 - q * (\theta_2)] \sum_{k=1}^{\infty} k [q * (\theta_2)]^{k-1} \\
 &\quad - \left(\frac{c^2\theta_2}{\theta_1-\theta_2} \right) \left(\frac{1+R^2}{(1-R)^2} \right) [1 - q * (\theta_1)] \sum_{k=1}^{\infty} k^2 [q * (\theta_1)]^{k-1} \\
 &\quad + \left(\frac{c^2\theta_2}{\theta_1-\theta_2} \right) \left(\frac{1+R}{1-R} \right) [1 - q * (\theta_1)] \sum_{k=1}^{\infty} k [q * (\theta_1)]^{k-1} \\
 &= \left(\frac{c^2\theta_1}{\theta_1-\theta_2} \right) \left(\frac{1+R^2}{(1-R)^2} \right) \frac{[1+q*(\theta_2)]}{[1-q*(\theta_2)]^2} + \left(\frac{c^2\theta_1}{\theta_1-\theta_2} \right) \left(\frac{1+R}{1-R} \right) \frac{1}{[1-q*(\theta_2)]} \\
 &\quad - \left(\frac{c^2\theta_2}{\theta_1-\theta_2} \right) \left(\frac{1+R^2}{(1-R)^2} \right) \frac{[1+q*(\theta_1)]}{[1-q*(\theta_1)]^2} + \left(\frac{c^2\theta_2}{\theta_1-\theta_2} \right) \left(\frac{1+R}{1-R} \right) \frac{1}{[1-q*(\theta_1)]}
 \end{aligned}$$

$$\begin{aligned}
&= \left(\frac{a^2 \theta_1}{\theta_1 - \theta_2} \right) \frac{(1+R^2)[1+q^*(\theta_2)]}{[1-q^*(\theta_2)]^2} - \left(\frac{a^2 \theta_1}{\theta_1 - \theta_2} \right) \frac{(1+R)(1-R)}{[1-q^*(\theta_2)]} \\
&\quad - \left(\frac{a^2 \theta_2}{\theta_1 - \theta_2} \right) \frac{(1+R^2)[1+q^*(\theta_1)]}{[1-q^*(\theta_1)]^2} - \left(\frac{a^2 \theta_2}{\theta_1 - \theta_2} \right) \frac{(1+R)(1-R)}{[1-q^*(\theta_1)]} \\
&= \left(\frac{\theta_1}{\theta_1 - \theta_2} \right) \frac{a^2}{[1-q^*(\theta_2)]^2} \{ (1+R^2)[1+q^*(\theta_2)] + (1-R^2)[1-q^*(\theta_2)] \} \\
&\quad - \left(\frac{\theta_2}{\theta_1 - \theta_2} \right) \frac{a^2}{[1-q^*(\theta_1)]^2} \{ (1+R^2)[1+q^*(\theta_1)] + (1-R^2)[1-q^*(\theta_1)] \} \\
&= \left(\frac{\theta_1}{\theta_1 - \theta_2} \right) \frac{a^2}{[1-q^*(\theta_2)]^2} \{ 2 + 2R^2 q^*(\theta_2) \} \\
&\quad - \left(\frac{\theta_2}{\theta_1 - \theta_2} \right) \frac{a^2}{[1-q^*(\theta_1)]^2} \{ 2 + 2R^2 q^*(\theta_1) \} \\
&= \frac{2a^2}{\theta_1 - \theta_2} \left[\frac{\theta_1(1+R^2 q^*(\theta_2))}{[1-q^*(\theta_2)]^2} - \frac{\theta_2(1+R^2 q^*(\theta_1))}{[1-q^*(\theta_1)]^2} \right]
\end{aligned}$$

For the product of X_1 and X_2 , the density function is

$$\begin{aligned}
q(x) &= \int_0^x g(u)h(x-u) du = (\alpha_1 \alpha_2 (e^{-(\alpha_2 x)} - e^{-(\alpha_1 x)})) / (\alpha_1 - \alpha_2) \\
q^*(\theta_1) &= \int_0^\infty [\alpha_1 \alpha_2 (e^{-(\alpha_2 x)} - e^{-(\alpha_1 x)}) / (\alpha_1 - \alpha_2)] e^{-(\theta_1 x)} dx \\
&= (\alpha_1 \alpha_2 / (\alpha_1 - \alpha_2)) [1/(\theta_1 + \alpha_2) - 1/(\theta_1 + \alpha_1)] \\
&= \alpha_1 \alpha_2 / ((\theta_1 + \alpha_1)(\theta_1 + \alpha_2))
\end{aligned}$$

Also,

$$q^*(\theta_2) = (\theta_2 + \alpha_1) / (\theta_2 + \alpha_2) \alpha_1 \alpha_2$$

Equation yields

$$E(W) = (\alpha / (\theta_1 - \theta_2)) [\theta_1(\theta_2 + \alpha_1)(\theta_2 + \alpha_2) / [\theta_2^2 + \theta_2(\alpha_1 + \alpha_2)] - \theta_2(\theta_1 + \alpha_1)(\theta_1 + \alpha_2) / [\theta_1^2 + \theta_1(\alpha_1 + \alpha_2)]]$$

Equation becomes

$$\begin{aligned}
E(W^2) &= \frac{\left(\frac{\{\theta_1\}}{\{\theta_1 - \theta_2\}} \right) \{ 2a^2 [(\theta_2 + \alpha_1)(\theta_2 + \alpha_2)]^2 \}}{\left\{ [(\theta_2 + \alpha_1)(\theta_2 + \alpha_2) - \alpha_1 \alpha_2]^2 \right\} \left\{ \frac{\{(\theta_2 + \alpha_1)(\theta_2 + \alpha_2) + R_{1\alpha_2}^{2\alpha}\}}{\{(\theta_2 + \alpha_1)(\theta_2 + \alpha_2)\}} \right\}} \\
&\quad - \left(\frac{\{\theta_2\}}{\{\theta_1 - \theta_2\}} \right) \{ 2a^2 [(\theta_1 + \alpha_1)(\theta_1 + \alpha_2)]^2 \} \left\{ \frac{(\theta_1 + \alpha_1)}{(\theta_1 + \alpha_2)} \right. \\
&\quad \left. - \alpha_1 \alpha_2 \right\} \left\{ \frac{\{(\theta_1 + \alpha_1)(\theta_1 + \alpha_2) + R_{1\alpha_2}^{2\alpha}\}}{\{(\theta_1 + \alpha_1)(\theta_1 + \alpha_2)\}} \right\} \\
&= \left(\frac{\{\theta_1\}}{\{\theta_1 - \theta_2\}} \right) \left\{ \frac{2a^2 (\theta_2 + \alpha_1)}{(\theta_2 + \alpha_2)[(\theta_2 + \alpha_1)(\theta_2 + \alpha_2) + R_{1\alpha_2}^{2\alpha}]} \right\} \left\{ \frac{R}{[\theta_2^2 + \theta_2(\alpha_1 + \alpha_2)]^2} \right\}
\end{aligned}$$

A numerical example

Changes to the parameters allow for numerical analysis of the analytical expressions of the expected and variance of the time to recruitment. By changing the mean of the interdecision durations (ii) changing α_1 -the loss of manhours of grade I and (iii) Table changing α_2 -the loss of manhours of grade II, the anticipated time to recruitment and its variation are computed and shown.

The expected time and variance of the time to recruitment for different values of the parameters θ_1 and θ_2 are respectively, and they increase as the mean of the interdecision times and the loss of manhours of grades I and II increase. As the values of the grade I and grade II thresholds grow, the predicted time to recruitment and the variance of the time to recruitment decrease respectively.

Conclusion

Dynamic manpower planning models provide a powerful mathematical framework for aligning workforce capabilities with long-term organizational objectives in an increasingly complex and uncertain environment. By integrating tools such as optimization, stochastic processes, system dynamics, and forecasting techniques, these models move beyond static headcount planning to support adaptive, data-driven decision-making. They enable organizations to anticipate future skill requirements, manage workforce flows, and balance costs, productivity, and sustainability over time.

From a strategic perspective, dynamic models facilitate sustainable organizational growth by ensuring that human capital evolves in tandem with market demands, technological change, and demographic shifts. They allow decision-makers to evaluate multiple scenarios, assess risks, and design robust policies for recruitment, training, promotion, and retention. This forward-looking capability reduces inefficiencies, mitigates talent shortages or surpluses, and enhances organizational resilience.

Moreover, the mathematical rigor underlying dynamic manpower planning strengthens transparency and accountability in workforce decisions. By explicitly modeling constraints, objectives, and trade-offs, these frameworks support evidence-based planning and continuous improvement. When embedded within organizational information systems and complemented by managerial judgment, they become practical tools for long-term human resource sustainability.

In conclusion, dynamic manpower planning models represent a critical bridge between quantitative analysis and strategic human resource management. Their effective implementation can help organizations achieve sustainable growth by optimizing workforce structures, fostering adaptability, and ensuring that human capital remains a core driver of competitive advantage in a rapidly changing world.

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