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RELIABILITY ANALYSIS OF A STOCHASTIC SYSTEM WITH MAINTENANCE OF STANDBY UNIT AND REPLACEMENT TIME AFTER INSPECTION FOR FAILED UNIT BY RPGT

Anju Dhall*

ABSTRACT

In this paper, reliability model of two identical units with cold standby system is analyzed stochastically. Maintenance policy for the standby has been adopted before getting it into operation. A system model is developed in which standby unit undergoes for maintenance with some probability when it is not operable at the failure of primary unit. The concept of inspection policy has also been introduced for detecting the need of repair or replacement before the failed unit repaired by some repair mechanism. There is a single repair facility which acts as inspection, replacement, maintenance and repair. The maintenance and repair of the unit are perfect. All random variables are statistically independent. The analysis is carried out on the supposition that all time distributions are general except failure, replacement and inspection time distributions are exponentials. Stochastic behavior of the system has been studied in steady state using semi-Markov process and regeneration point graphical technique and several parameters of interest are obtained. Numerical results pertaining to some special cases are also added. The behavior of some important performance measures have been examined graphically giving particular values to the parameters.

Keywords: *Stochastic System, Maintenance and Inspection Policies, Repair and Reliability Models.*

Introduction

A system with high reliability can be achieved by the use of redundancy and maintenance. Such two unit redundant systems are basic in reliability theory and can be found in practical application. Therefore, several research papers on reliability measures of redundant systems have been written by the scholars including Gupta and Mumtaz (1996) and Yadavalli et al. (2004).It is an important problem to operate a system in specified long time without failure. We have known some policies to maintain a system. In particular, we make the system redundant and maintainable. In recent paper, we have discussed a two unit redundant system with maintenance of standby unit & inspection for replacement time for failed unit. . On the other hand, readiness of standbys is of dynamic consequence as these can be brought into operation at the failure of operating unit. Thus, in such a situation, maintenance of the standby unit becomes necessary before getting it into operation and this fact can be exposed by inspection.

Recently, Malik and Barak (2013) Malik (2013) and Yusuf (2014) evaluated performance measures of cold standby systems with maintenance and repair.

While considering these practical situations in mind, the purpose of the present study is to analyze reliability model of a stochastic system of two identical units - primary unit is initially operative and the other is kept as spare in cold standby. There is a single repair facility which acts as inspection,

* Department of Mathematics, G.G.D.S.D. College, Palwal, Haryana, India.

replacement, maintenance and repair. A system model is developed in which standby unit undergoes for maintenance with some probability when it is not operable at the failure of operative unit. According to policy, server first inspects the failed unit to see the feasibility of its operation at the failure of operative unit. And, if failed unit is not found fit for operation, then the server starts its replacement immediately. The unit works as new after maintenance and repair. The failure time of the unit follows negative exponential distribution where the distributions of maintenance, inspection and repair times are taken as arbitrary with different probability density functions. All random variables are statistically independent. Switch devices are perfect.. The analysis is carried out on the supposition that all time distributions are general except failure, replacement and inspection time distributions are exponentials. Stochastic behavior of the system has been studied in steady state using semi-Markov process and regeneration point technique and several parameters of interest are obtained. Numerical results pertaining to some special cases are also added. The behavior of some important performance measures have been examined graphically giving particular values to the parameters.

Notation

O/Cs	The unit is in operative/ cold standby mode
λ	Constant failure rate of the unit.
$a/b/a_1/b_1$	The probability that standby unit is found fit for operation/non-operable.
SU_m	The rate Standby unit under maintenance.
$g(t)/G(t)$	pdf/cdf of repair time .
$h(t)/H(t)/h_1(t)/H_1(t)$	pdf/cdf of inspection time of the unit.
$m(t)/M(t)$	pdf/cdf of preventive maintenance time.
FUr/FWr	The unit is failed and under/waiting for repair continuously from previous state.
FUi/FUI	The unit is failed and under/waiting for inspection continuously from previous state.
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of direct transition time from a regenerative state regenerative state(0,t].
$q_{ij.k}(t)/Q_{ij.k}(t)$	pdf/cdf of first passage time for a regenerative state S_i to regenerative state S_j or to failed state S_j visiting state S_k once in (0,t]
$W_i(t)$	Probability that the server is busy in state S_i up to time without making transition to any other regenerative state or returning to the same via one or more regenerative states.
$M_i(t)$	Probability that the system is up initially in state $S_i \in E$ is up at the time "t" without visiting to any other regenerative state.
μ_i	The mean sojourn time spent in state $S_i \in E$ before transition to any other state.
μ_i'	The total unconditional time spent in state before transition to any other regenerative state given that the system entered regenerative state i at time t=0
f_i	Fuzziness measure of the i-state
n_i	Expected time spend while doing a job,given that the system entered regenerative state i at time t=0
$V_{kk}/\overline{V_{kk}}$	Transition probability factor of reachable state k of the $k\ cycle/\overline{k\ cycle}$
$i \xrightarrow{r} j$	Rth directed simple path from i state to j state,r take +ve integral values from i state to j state
$\xi \xrightarrow{sff} i$	A directed simple failure free path from ξ state to i state
m_{ij}	Contribution to mean sojourn time in state S_i when system transits directly to state S_j(S_i,$S_j \in E$) $\mu_i = \sum m_{ij}$ so that $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$

Ⓢ/©	Laplace Stieljes convolution/ Laplace convolution
~/*	Symbol for Laplace Stieljes transform (LST)/ Laplace transform (LT
'	Symbol for derivative of the function
cycles	$L_1 = (1,0,2,1); L_2 = (1,3,6,1); L_3 = (2,6,2); L_4 = (1,4,5,1)$ And $(i,j,k) = (i,j)(j,k)$

The possible transition states of the system shown in fig.1

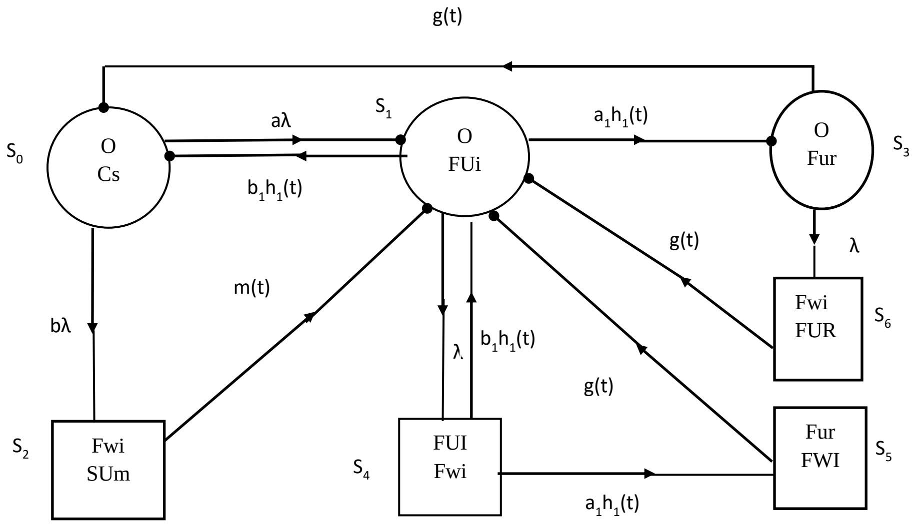

Fig.1

● Regenerative Point 0 up state ▭ failed state

Transition Probabilities and Mean Sojourn Times

Simple probabilistic consideration yield the following expressions the non-zero elements p_{ij} are given by

$$p_{01} = p_{02} = p_{10} + p_{14} + p_{13} = p_{10} + p_{11.3} + p_{14} = p_{30} + p_{36} = p_{41} + p_{45} = p_{51} = p_{61} = 1$$

The mean sojourn times (μ_i) in the state S_i are

$$\mu_0 = \int_0^\infty P(T > t)dt = \mathrm{m}_{01} + \mathrm{m}_{02} = \frac{1}{\lambda}, \mu_1 = \mathrm{m}_{10} + \mathrm{m}_{14} + m_{13} = \frac{1 - h^*(\lambda)}{\lambda}$$

$$\mu_2 = \mathrm{m}_{21} = -m^*(0), \mu_3 = \mathrm{m}_{30} + m_{36} = \frac{1 - g^*(\lambda)}{\lambda}$$

$$\mu_4 = \mathrm{m}_{41} + \mathrm{m}_{45} = -h_1^{*'}(0) \mu_5 = \mathrm{m}_{51} = -g^{*'}(0) = \mu_6 = \mathrm{m}_{61}$$

$$\mu_1' = \mathrm{m}_{10} + \mathrm{m}_{11.4} + m_{11.45} + m_{13}$$

$$\mu_3' = \mathrm{m}_{30} + \mathrm{m}_{31.6} + m_{22.6} + m_{23.7}$$

Mean Time to System Failure

The regenerative un-failed states to which the system can transit before entering any failed state are i=0,1,3;k_1,k_2=Nil

$$\text{MTSF} = \left[\sum_{i,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r(sff)} i)\right\}.\mu_i}{\prod_{k1\neq\xi}\{1-V(\overline{k1,k1})\}}\right\}\right] \div \left[1-\sum_{s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r(sff)} \xi)\right\}}{\prod_{k2\neq\xi}\{1-V(\overline{k2,k2})\}}\right\}\right]$$

$$\text{MTSF} = \frac{(0-0)+(0-1)+(0-3)}{1-(0,1,0)-(0,1,3,0)} = \frac{\text{N}_1}{\text{D}_1} \;; N_1 = \mu_0 + p_{01}\mu_1 + p_{01}p_{13}\mu_3 \;\;, D_1 = 1 - p_{01}p_{10} - p_{01}p_{13}p_{30}$$

Steady state Availability

The regenerative state at which system is available are i=0,1,3, and j=0,1,,3.Base state=1:

$$A_\xi = \left[\sum_{j,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} j)\right\}f_j.\mu_j}{\prod_{k1\neq\xi}\{1-V(k1,k1)\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} i)\right\}.\mu_i^1}{\prod_{k2\neq\xi}\{1-V(k2,k2)\}}\right\}\right]$$

$$A_2 = \frac{\{(1,0)+(1,3,0)\}\mu_0 + (1-1)\mu_1 + (1-3)\mu_3}{1-(0,1,0)-(0,1,3,0)} = \frac{\text{N}_2}{\text{D}_2} \;; N_2 = (\mu_0 + \mu_1 + p_{13}\mu_3) \;\;, D_2 = (\text{p}_{13}p_{30} + p_{10}) + \mu_1' + p_{13}\mu_3'$$

Busy Period of the Server due to Inspection and maintenance

The regenerative state where the server is busy while doing inspection/maintenance is i=1 and j=0,1,3

$$B_\xi = \left[\sum_{j,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} j)\right\}\eta_j}{\prod_{k1\neq\xi}\{1-V(k1,k1)\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} i)\right\}.\mu_i^1}{\prod_{k2\neq\xi}\{1-V(k2,k2)\}}\right\}\right]$$

$$B_0 = \frac{N_3}{D_2}; N_3 = W_1^{im}(0)\text{and}\text{D}_2 \text{ is already defined.}$$

$$W_1^{im}(t) = e^{-\lambda t}\overline{H(t)} + [\lambda e^{-\lambda t} \odot 1_]\overline{H(t)}$$

Busy Period of the Server Due to Repair

The regenerative state where the server is busy while doing repair is i=3and j=0,1,3

$$B_\xi^r = \left[\sum_{j,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} j)\right\}\eta_j}{\prod_{k1\neq\xi}\{1-V(k1,k1)\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} i)\right\}.\mu_i^1}{\prod_{k2\neq\xi}\{1-V(k2,k2)\}}\right\}\right]$$

$$B_0^r = \frac{N_4}{D_2}$$

where

$N_4 = P_{13}W_3^{r*}(0)$ and D_2 is already specified

$$Where\; W_3^{r^*}(t) = e^{-\lambda t}\overline{H(t)} + [\lambda e^{-\lambda t} \odot 1_]\overline{H(t)}$$

Expected number of visits of the server:

Let $N_i(t)$ be the expected number of visits by the server at instant't' given that the system entered regenerative state S_i at t=0. The recursive relations for $N_i(t)$ are:

$$V_\xi = \left[\sum_{j,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} j)\right\}}{\prod_{k1\neq\xi}\{1-V(k1,k1)\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\left\{pr(\xi \xrightarrow{s_r} i)\right\}.\mu_i^1}{\prod_{k2\neq\xi}\{1-V(k2,k2)\}}\right\}\right]$$

$$V_0 = N_4 \div D_2$$

$$N_5 = P_{01}(p_{10} + p_{13}p_{30})$$

and D_2 is already defined.

Cost –Benefit Analysis:

The profit incurred to the system in steady state is given by

$$\text{Profit} = K_0A_0 - K_1B_0 - K_2B_0^r - K_3V_0$$

Where

K_0 =Revenue per unit up-time of the system.

K_1 =Cost per unit time for where server is busy due to inspection / maintenance.

K_2 = Cost per unit time for where server is busy due to repair.

K_3 = Cost per unit time for where server is visited.

Conclusion

There is a vertical deterioration in mean time to system failure (MTSF) with the increase of failure rate (λ) for other fixed parametric values as shown in figure 2. Figure 3 indicate that Availabilty of the system model go on decreasing as failure rate (λ) increases. Figure 4 indicate that profit of the system model go on decreasing as failure rate (λ) increases. All performance measures has direct relation with repair rate, rate of inspection, and rate when standby unit goes for maintenance. However, system becomes more profitable when replacement rate increase in comparsion with repair .Particularly it is concluded that the system becomes more profitable by making replacement provided unit has more chances of maintenance in cold standby mode.

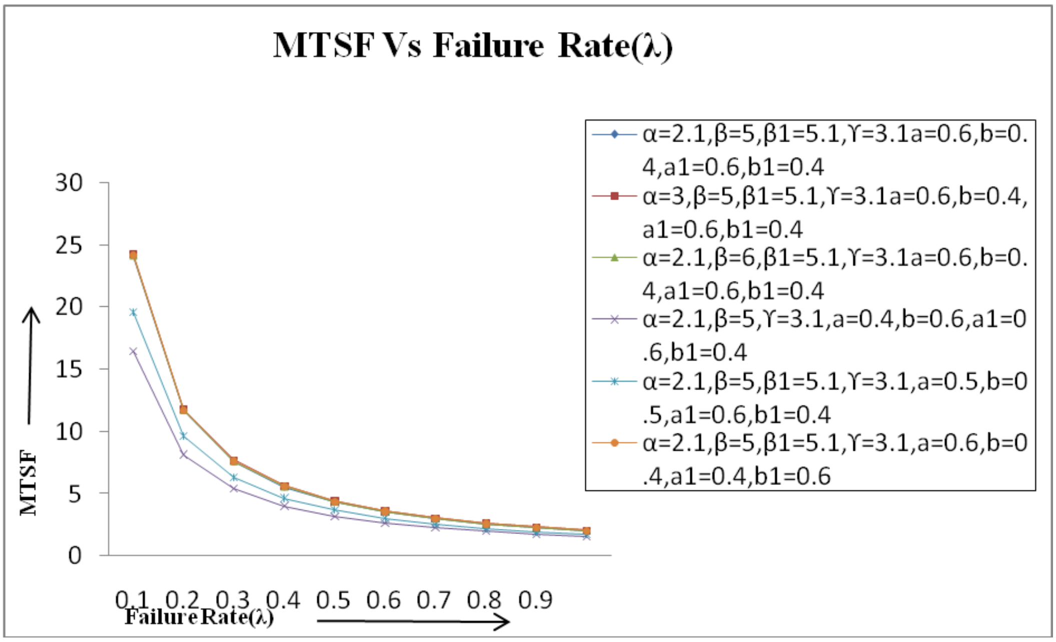

Fig.2

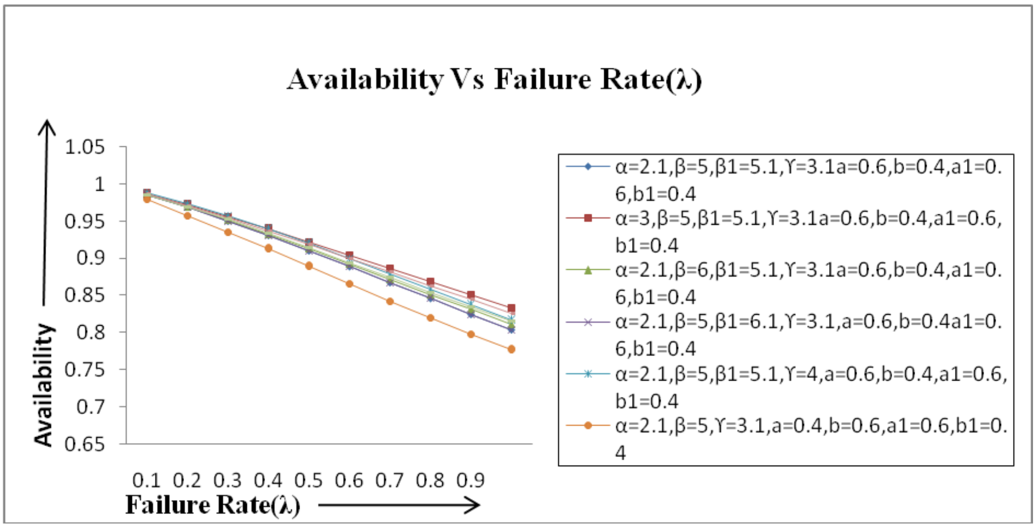

Fig.3

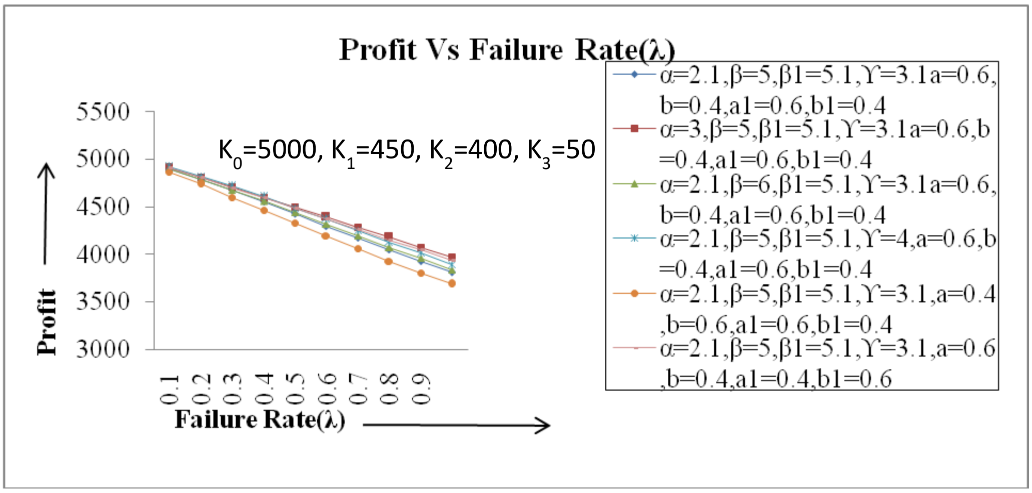

Fig.4

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