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A SIGNIFICANT ROLE OF MATHEMATICAL APPROACH IN FACE RECOGNITION: A STUDY

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ABSTRACT

In this research paper, I discuss the different mathematical approaches that apply to face recognition. I know that every human being has unique poses, facial expressions, hair style, pictorial style and light conditions. So, face recognition is a very difficult task. To solve this, I need to test different image recognition methods such as image processing tools and heuristics. In this research paper I have explored the main features of the mathematical approach.

Keywords: *Face Recognition System, Eigenfaces, Key-Frame, Colour Histogram*.

Introduction

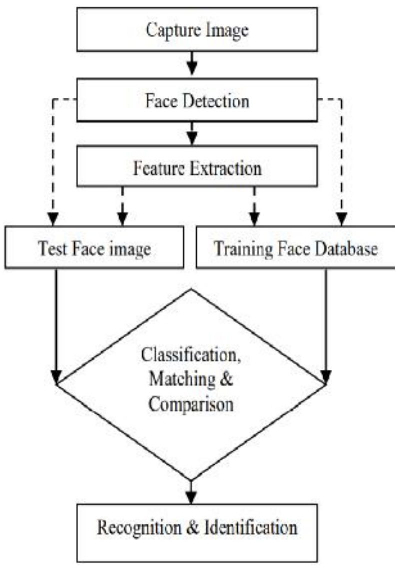

Figure 1: The Eigenface approach. Sample training faces are revealed (left), followed by the first fifteen principal Eigenfaces.

The Face recognition, the art of matching faces from a facial database, is a non-intrusive biometric method dating back to the 1960s. In earlier attempts, people have tried to understand which facial features help us perform simple tasks of identification, such as identifying a person, deciding a person's age and gender. Doing, and classifying facial expressions and even beauty. Amazed by Devenchi's Mona Lisa smile, for example - what makes it so special? Research in this area has fascinated psychologists, behavioural scientists, artists, and, more recently, mathematicians and engineers. In this note we describe some of the basic tools of mathematics.

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Most face recognition algorithms are found in one of two main groups: feature-based and image-based algorithms. Feature-based methods explore a set of geometric features, such as the distance between the eyes or the size of the eyes, and use these steps to represent a given face. These features are done using straightforward filters with the expected templates. These methods are somewhat invasive to changes in lighting and may partially compensate for changes in camera position. Though, they are searching to aging and face expressions. It is also not clear what features are important for classification, an area that requires more study of mathematics. There are basic mathematical results in the literature that try to solve these questions and have not yet been fully exploited for face recognition. In fact, the first documented work on computerized face recognition - by Bladeso, more than 40 years ago - was based on these ideas. The image-based system, the second central approach to facial recognition, is based on theories such as eigenfaces (see Fig: 1). After assembling a large training set of images, component analysis is used to count the eigenfaces phases. After that, each new face is projected into the character of Principal eigenfaces. This approach to small changes, both externally and internally, is highly sensitive, although it is still one of the most popular methods in the industry. Well-known research in this area was carried out by Turk and Pentlands.

There are two foremost types of face recognition systems. The first step is to check if the person in front of the camera is a member of a prohibited group (20-500 people). Typically, such systems are used to gain control of buildings, computers, etc. The feature of such systems is the real-time response and checking for changes in the status and appearance of the person sensitivity.

Other types of systems identify a person by a photo search in a large database or confirm their absence. Such a system contains 1.000-1.000.000 images with a database. It can work offline. We try to design another type of system.

Our Approach

There are approximately different approaches to create such a system:

- Eigenface analysis
- Template matching
- Graph matching
- Neural network-based approach and
- Many others

Most of these faces are considered as flat surfaces and the difference in the familiarity of the faces of the competition is ignored. In fact, a face is a 3D convex object that has the ability to rotate and change shape. The biggest difficulty in recognizing the face is to formulate a strong feature for the unique expression of the human face.

Eigenface Analysis

Face images are presented in place of a feature that best encodes the differences in known face images. Face space is defined by eigen faces, which are the eigenvector of the set of faces.

- ***Eigenfaces*** **Method:** The origin of the eigenfaces method is principal component analysis (PCA). Eigenfaces and PCA have been used by Syrovich and Kirby to effectively represent face images. He started with a bunch of original face images, and calculated the best vector system to pressure immigration. Then Turkey and Pentland used eigenfaces to deal with the problem of identity. Analysis of principal components is a method of projection into a subspace and is widely used in pattern identification. One of the purposes of PCA is to convert large-dimensional mutual vectors with small-dimensional unconnected vectors. The second purpose is to calculate the basis of a set of data. The main advantages of PCA are its low sensitivity to noise, reducing memory and capacity requirements and increasing efficiency due to its operation on small dimensions.
- **Mathematical Model:** A mathematical model describes mathematical concepts and language systems. A model can help study the effects of different components of the mathematical modelling for our system is as follows:

S= {∑, F, δ, C}

S = Face Recognition.

∑ = set of input symbols = {Video File, image, character information}

F = set of output symbol = {Match Found then notification to user, Not Found}

- δ = Start
- Read training set of images $N \odot N$ images
- Resize image dimensions to $N^2 \odot 1$
- Select training set of $N^2 \odot M$

Dimensions, M: number of sample images

- Find the avg. face, subtract from the faces in the training set,

create matrix A

$$\Psi = \frac{1}{M}\sum_{i=0}^{M} \Gamma i$$

Where,

Ψ= average image,

M= number of images, and

Γi= image vector.

$\Phi i = \Gamma i - \Psi$

Where, i = 1, 2, 3, …, M.

A = [Φ1, Φ2, Φ3…ΦM]

- Calculate covariance matrix: AA'

$$C = A^T \Phi A$$

- Calculate eigenvectors of the c covariance matrix.
- Calculate eigenfaces = No. of training images –no. of classes (total number of people) of eigenvectors.
- Create reduced eigenface space

The carefully chosen set of eigenvectors are multiplied by the A matrix to create a reduced eigenface

- Analyse eigenface of image in question.
- Calculate Euclidian aloofness between the image and the eigenfaces.
- Find the minimum Euclidian distance.
- Output: image with the minimum Euclidian distance or image unrecognizable

C = {The system will 'not process the audio data, Eigenfaces determination generate the grayscale images, the algorithm will run only on key frames.

- **Template matching:** This is a digital image processing technique for finding small parts of an image that resemble a template image. It can be used as part of quality control in manufacturing, as a way for mobile robots to navigate, or as a way to detect edges in images.

The main challenges in template matching work are: detecting irrational changes, light and background changes, background clutter and scale changes. One basic method of matching a template is using an image patch (template), tailored to a particular feature of the search image we want to search for. This procedure can be easily achieved on grey images or edge images. The cross-linking output will be highest in places where the image structure matches the mask structure, where the values of the large image are multiplied by the values of the large mask.

This method is usually applied by selecting a portion of the search icon to use as the first template: we will call the search image **S (x, y),** where **(x, y)** the search represents the coordinates of each pixel in the image. We will call the template **T (X t, Y t)**, where **(Xt, Yt)** represent the points of each pixel in the template. We then basically move the centre (or the origin) of the template $\mathbf{T(x_t, y_t)}$ over each **(x, y)** point in the exploration image and calculate the sum of products between the coefficients in **S(x, y)** and $\mathbf{T(x_t, y_t)}$ over the entire area spanned by the pattern. Since all possible positions of the template are considered in terms of the search image, a position with a high score is the best position. This method is sometimes called "Linear SpatialFiltering" and the template is called a filter mask.

One more way to deal with transformation problems on images using template matching is to compare the intensity of the pixels using the SAD (combination of absolute differences) measurement.

The intensity of a pixel in a search image with coordinates $\mathbf{(x_s, y_s)}$ has intensity $\mathbf{I_s(x_s, y_s)}$ and a pixel in the template with coordinates $\mathbf{(x_t, y_t)}$ has intensity $\mathbf{I_t(x_t, y_t)}$. Consequently, the absolute difference in the pixel intensities is well-defined as $\mathbf{Diff(x_s, y_s, x_t, y_t) = | I_s(x_s, y_s) - I_t(x_t, y_t) |}$.

$$SAD(x,y) = \sum_{i=0}^{T_{rows}} \sum_{j=0}^{T_{cois}} \mathrm{Diff}(x+i, y+j, i, j)$$

The mathematical representation of the idea of looping through pixels in the search image when we translate the original of the template on each pixel and measure SAD is:

$$\sum_{x=0}^{S_{rows}} \sum_{y=0}^{S_{cols}} \mathrm{SAD}(x,y)$$

$\mathbf{S_{rows}}$ and $\mathbf{S_{cols}}$ signify the rows and the columns of the search image and $\mathbf{T_{rows}}$ and $\mathbf{T_{cols}}$ signify the rows and the columns of the template image, separately. In this method the lowermost SAD score gives the approximation for the best position of template within the search image. The method is simple to contrivance and recognize, but it is one of the measured methods.

- **Graph Matching:** Ladies, Werbergen, Boehmen, Lange, Malsburg, Wortz, and Konen proposed an animated link structure for distorted inherent object recognition.[11] Flexible group graph matching is used to find the nearest storage graph by estimating the set of features. In them, the feudal points on the face are represented by a set of parasitic components (jets). The flexible group graph matching algorithm marks the first faces of the novel. Both localization and comparison, Gaber jets are flown to historic sites. In localization, the jets are extracted from the novel images and matched with the jets extracted from the seats of the training / model jets. The similarities between the images in the novel are illustrated by the similarities between the local Gabor jets found in the facial expressions.

The graph corresponds to an unstructured graph and an object that has nodes at the fiducial point of the facial image. The graph structure is the same for each face. However, some nodes may be unexplained due to the connection. The face flag graph is a stack-like structure consisting of a jet group instead of a jet. Each node is labelled with a cluster of jet planes, and the facial patterns on each edge are labelled with the average distance between the same nodes. For a particular pose, assume that the M model graphs of the same structure are G^{BM} (m = 1,2,…, M). The corresponding FBG B is then given the same structure, its nodes are labelled with groups of jets J_n^{Bm} and its edges averaged distances $\Delta\vec{x}_e^B = \sum_m \Delta\vec{x}_e^B/M$. Graph similarity between an image graph and 1,2, … ,N and edges e = 1,2, … , E and an FBG B with model graphs m = 1,2, … M the resemblance is well-defined as:

$$S_B(G^I, B) = \frac{1}{N}\sum_n max\,(S_\varphi(J_n^I, J_n^{Bm})) - \frac{\lambda}{E}\sum_e \frac{(\Delta\vec{x}_e^I - \Delta\vec{x}_e^B)^2}{(\Delta\vec{x}_e^B)^2},$$

λ Determines the importance of the ratio of jets to metrics. J_n are the jets at nodes n, and $\Delta\vec{x}_e$ are the distance vectors used as labels at edges? EBGM uses a distorted facial structure and integrates these points so that the graphic graph can be translated, scaled, rotated, and distorted into the image plane. EBGM is less likely to have a missing feature and will still be able to identify images that have some features unlike the Eigenface and Fisher faces that have a change or are missing that both features are missing feature. The identification rates for the 15-degree and 30-degree rotation match tests of 111 faces are 86.5% and 66.4%, respectively. In general, EBGM performs better than other identification techniques in terms of rotation. However, the matching process is computationally expensive.

- **Neural Networks:** Nerve networks are non-linear networks that are suitable for representing non-linear faces. Neural networks are used in many applications such as pattern recognition problems, character recognition, object recognition, and so on. The main purpose of the neural network is to capture the complex facial features. Multi-Layer Perceptron (MPL) with feed forward learning algorithm was applied for facial recognition. T.J Stonham made his first attempt at face recognition using a single-layer adaptive network called WISARD, which has a separate network for each stored individual. S. Lawrence, Giles, Tsoi, and A.D. Back have proposed a hybrid neural network that combines local image sampling, self-mapping (SOM) neural networks, and a virtual neural network. SOM provides the amount of sample images, thus minimizing the dimensions and conveying the message. The SOM consists of N nodes ordered in a two-dimensional lattice structure and each node has 2 or 4 neighbouring nodes, respectively. Typically, a SOM's life cycle consists of three phases: the learning phase, the training phase, and the testing phase. During the learning phase, the neuron weighing near the input data vector is declared the winner.

  Learning algorithm may be summarized as follows:

  - **Initialization:** Select random values for the initial vector weights $w_j(0)$,

  for j=1,2,…,I where I is the total number of neurons

  $w_i = [w_{i1}, w_{i2}, w_{i3} \ldots\ldots w_{il}]^T \in \Re^n$

  - ***Sampling:*** Draw a sample x from the input space with a confident prospect.

  $x = [x_1, x_2, \ldots\ldots\ldots x_l]^T \in \Re^n$

  - **Similarity Matching:** Find the best matching (winning) neuron i(x) at time t, $0 < t \leq n$ by using the minimum distance Euclidean criterion:
  - **Updating:** Adjust the synaptic weight vector of all neurons by using the bring up-to-date

    $$i(x) = \arg min \min_j \|x(n) - w_j\|, j = 1,2,\ldots,l$$

    formula:

    $$w_j(n+1) = w_j(n) + \eta(n) h_{j,i(x)}(n)\,(x(n) - w_j(n))$$

  Where $\eta(n)$is the learning rate parameter, and □$j,i(x)$ is the neighbourhood function cantered around the winning neuron i(x).

- **Repeat from** step 2 until no changes in the feature map are observed.

The self-generated map recorded two values: the total number of winning times for both titles and the image is not in the database. During the training phase, the number of wins is also recorded with the input sample label for each node. During the testing phase, each input vector is compared with all the nodes in the SOM, and some of the best matches are found based on a minimum distance calculator.

Face recognition, which uses features derived from the isolated cosine transform (DCT) coefficient with SOM-based classification, was performed using an image database of 25 face images of each of the 5 subjects. After training for 850 positions, he achieved an identification rate of 81.36% for 10 consecutive trials. There are 5 different facial expressions.

Conclusion

I have suggested face recognition using different algorithms with a mathematical approach. In this research paper I have tried to explain the mathematical approach to face recognition. Face recognition has been active for many years and many researchers and authors are now looking for new things to do in this field for included in this section. It has a large number of commercial applications. However, the original request is for law and order and national identity measures. The techniques under discussion are being researched to find a better algorithm that will perform at higher accuracy and less computational time.

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