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MODEL ORDER REDUCTION OF LINEAR DYNAMIC SYSTEM BY USING BASIC CHARACTERISTICS AND ROUTH ARRAY METHOD

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ABSTRACT

The authors propose a hybrid method for reducing the order of the high order dynamic systems. In the proposed method, the denominator polynomial of the reduced order model is obtained by using the basic characteristics of the original system, which are maintained in the reduced model while the coefficients of the numerator are obtained, by using Routh array method. This method is fundamentally simple and generates stable reduced models if the original high- order system is stable. The proposed method is illustrated with the help of the numerical example taken from the literature.

Keywords: *Model Order Reduction Basic Characteristics, Routh Array, Stability, Transfer Function.*

Introduction

The approximation of linear systems have an important role in many engineering applications, especially in control system design, where the engineer is faced with controlling a physical system for which an analytic model is represented as a high order linear system. In many practical situations, a fairly complex and high order system is not only tedious but also not cost effective for on-line implementation. It is therefore desirable that a high system will be replaced by a low order system such that it retains the main qualitative properties of the original system. Several order reduction techniques for linear dynamic systems in the frequency domain are available the literature [1-4]. Further, some methods have also been suggested by combining the features of two different methods [5-7]. The Pade approximation method was originally introduced by Pade [8]. This method is computationally simple and fits initial time moments and matches the steady state values. The disadvantage of this Method is that the reduced model may be unstable even though the original system is stable. Sumit Mondal [9] utilizing the basic characteristics of original system and pade approximation to get reduced order system. Authors proposed that, the denominator polynomial of reduced model is obtained using basic characteristics such as undamped natural frequency of oscillations (ωn), damping ratio(£),settling time(Ts), peak overshoot(M) & peak time(tp) while the coefficient of numerator of reduced order system is obtained by using Routh array can be [24]. This method is computationally simple and is applicable to stable systems.

In the next section, the algorithm is described in detail with the help of numerical example.

Statement of the Problem

Let the transfer function of high order original system of the order ‘n’ be

G(s) = (a_0 s^(n-1)+a_2 s^(n-2)+ a_3 s^(n-3)+……+a_(n-1))/(s^(n)+b_1 s^(n-1)+b_2 s^(n-2)+b_3 s^(n-3)+ 〖…..+b〗_n) (1)

Where a; 0≤ i ≤ $n-1$ and bi; 0≤ i ≤ n known scalar constants.

〖R〗 k (s)=(c_0+c_1 s+c_2 s^2+⋯.+c_(k-1) s^(k-1))/(d_0+d_1 s+d_2 s^2+⋯.d_k s^k) (2)

Let the transfer function of the reduced model of the order ‘k’ be

cj ; 0≤ j ≤ $k-1$ and dj ; 0≤ j ≤ k are unknown scalar constants.

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The aim of this paper is to realize the kth order reduced model in the form of (2) from the original system (1) such that it retains the important features of the original high -order system.

Reduction Method

The reduction procedure for getting the kth-order reduced models consists of the following two steps:

- **Step 1:** Determination of the denominator polynomial for the kth-order reduced model using basic characteristics of original system by the following procedure

  Firstly determine the basic characteristics of original system

  Then assume damping ratio (ξ)=0.99 for an a periodic or almost periodic system, and number oscillations before the system settles=1

  Determine the natural frequency(ωn) using

  $t_s=4/(€*w_n)$

  Obtain the reduced order denominator as

  $D2(s) = s2+2*£*\omega n+ \omega n2$

- **Step 2**

The Routh array method is based on Routh stability criterion. In this method, none of the algorithm is required. This is a very simple procedure for reducing model for a high- order system. The array is usually written in the following form, from the coefficient of a given polynomial [6].

$G(s) = (a_1 s^{(n-1)}+a_2 s^{(n-2)}+ a_3 s^{(n-3)}+\ldots+a_n)/(s^{n}+b_1 s^{(n-1)}+b_2 s^{(n-2)}+b_3 s^{(n-3)}+ 〖\ldots+b〗_n)$ (3)

Where, n=2, 3, 4…………..

The routh array for the numerator and denominator polynomials of Eq.3. are shown in below in Table I and Table-II.

Table 1: Routh Stability Array For Numerator

(a_11 &a_12& a_13& a_14& -@a_21 &a_22& a_23& a_24& -@a_31 &a_32& a_33& -&@- &-& -& -&@a_(n,1) &-&&&@a_(n+1,1)&-&&&)

Table 2: Routh Stability Array For Dinominator

(b_11 &b_12& b_13& b_14& -@b_21 &b_22& b_23& b_24& -@b_31 &b_32& b_33& -&@- &-& -& -&@〖 b〗_(n-1,1) &-&&&@b_(n,1)&-&&&)

The first, third and fifth, etc. rows of each table consists of odd coefficients and the second, fourth and sixth, etc. rows of each table consist of even coefficients. The first two are generated from the given transfer function and remaining rows are calculated by below equation Eq.4

$C_{ij} =C_{(i-2,j+2)}- ((C_{(i-2,1)} \times C_{(i-1,j+1)}))/C_{(i+,1,1)}$

For $i\geq 3$ and $i\leq[(((n-1+3))/2)]$,

Here "n" refers to the order of the polynomials. So that the first two rows are made available for each table from given polynomials, then the other rows are easily reconstructed by this approach. In paper, this approach is used for finding reduced order numerator of given high-order transfer function.

Method for Comparison

In order to check the accuracy of the proposed method the relative integral square error ISE index in between the transient parts of the reduced models and the original system is calculated using Matlab / Simulink.

The integral square error ISE is defined as

$ISE=\int_0^\infty [y_n(t)-y_k(t)]^2 \, dt$

Numerical Example

The proposed method explains by considering numerical example, taken from the literature. The goodness of the proposed method is measured by calculating the rise time (tr), settling time (ts) and maximum overshoot (Mp) and compare with the original system.

Example: Consider a fourth order system from the literature

$G(s) = (24+24s+7s^2+s^3)/(24+50s+35s^2+10s^3+s^4)$ (4)

- **Step 1:** Determination of Denominator of reduced order

Denominator of reduced order model is determine using following basic characteristics of original system

Rise Time: 2.2603

Settling Time: 3.9308

Settling Min: 0.9019

Settling Max: 0.9990

Overshoot: 0

Undershoot: 0

Peak: 0.9990

Peak Time: 6.8847

£ =0.99 for an aperiodic or almost periodic system, and number oscillations before the system settles=1

Since ωn=4/(ε*ts)

Therefore ωn = 4/0.99*3.93=1.028

Using basic characteristic

D(s) = s2+2 x £ x ωn s+ ωn 2

=s2+2x0.99x1.028s+ (1.028)2

Step-2 Determination of numerator using Routh array

N(s) = 20.571s + 24

R(s) = (N(s))/(D(s)) = ((20.571s +24))/(s2+2.0354s+1.0568)

Applying steady state correction to reduced order model.

SSO = 24/24 = 1

SSR = 24/1.0568 = 22.71

K2 = SS0/SSR2 = 1/22.71 = 0.045

So, that the finally second order transfer function are shown below in eq.

R2(s) = (0.925s + 1.08)/(s^2+ 2.0354s+1.0568) (5)

Table 3: Qualitative Comparison with the Original System

System Overshoot	Rise Time (Ts)	Peak (Mp)	Settling Time (Ts)
Original System	2.602	0	3.9308
Reduced System	2.1427	0	3.7564

Table 4: Comparison of ISE

Method of Order Reduction	Reduced Models: R2 (S)	ISE
Proposed Method	24 + 20.57145s /1.0569 +2.0356s+s2	4.19 X 10-3
Chidambara	2-s2 / 2+3s+s2	220.2379X10-3
Davison	2-s2 / 2+3s+s2	220.2379X10-3
Krishnamur THY and Seshadri	24+20.5714s / 24+42s+30s2	9.5891X10-3
Gutmanet AL.	2[144+48s] / 288+300s +70s2	4.5593X10-3
SHIEH & WEI	2.3014 + s / 2.3014+5.7946s+s2	142.5607X10-3

Conclusion

The authors presented an order reduction method for the linear dynamic system of high order systems. The basic characteristics of original system is used for determination of denominator polynomial of the reduced model while Routh array is used for calculation of the numerator coefficients. The advantages of proposed method are stable, simplicity, efficient and computer oriented. The proposed method has been explained with an example taken from the literature. The step responses and Bode plots of the original and reduced system of second order are shown in the Figure-1 and Figure-2 respectively. A quantitative comparison of reduced order model obtain by proposed method with the original system is shown in the Table-I from which we can conclude that proposed method is comparable in quality. Table –II shows the comparison of ISE with other methods and it shows that proposed method has least ISE as compare to other method available in literature.

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