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WAITING TIME ANALYSIS OF QUEUEING SYSTEM HAVING COMBINATION OF SIX SERVERS CENTRALLY LINKED WITH A COMMON SERVER

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ABSTRACT

The present paper deals with the mathematical and graphical study of a queueing system in which five parellel servers are linked centrally with a common server in series. Waiting time of the whole system has been derived from mean queue length of a queueing system having combination of six servers centrally linked with a common server.

Keywords: *Centrally Linked Server, Steady State Equations, Generating Function Technique, Waiting Time.*

Introduction

Queue theory is concerned with the mathematical study of queues or waiting lines (seen in banks, post offices, hospitals, airports, etc.). The formation of waiting lines usually occurs whenever the current demand for a service exceeds the current capacity to provide that service.

In many cases the customer's arrival and his or her service time are not known in advance or can not be predicted accurately. Otherwise, the operation of the service facility could be scheduled in a manner that would eliminate waiting completely. Both arrival and departure phenomena are random. This necessitates mathematical modelling or queuing systems/ models to alleviate waiting. It involves reducing excessive costs that result from creating excess service capacity and at the same time ensuring that the system has enough service capacity to avoid long waiting lines. There has to be a balance between service capacity and waiting time. Therefore, an industry or an agency would like to provide such services and also maintain balance between the cost of service and the cost associated with waiting for the service.

Here our study is related to feedback queueing system in which six servers out of which one server is centrally linked with the other five servers in series. Kamal Datt et al. (2022) find the waiting time analysis of an hierarchical structured queuing system with feedback and revisit of customer at most once to any of the servers.

Harminder Singh et al (2019) worked on the analysis of network queue model comprised of parellel channels centrally linked with a common server.

Keeping this in view, Kumar and Taneja (2019), worked on the feedback queuing system comprising of three servers linked in series hierarchically in which a customer firstly join the first server, then either he/she may leave the system after getting the service or may move to the second higher ordered server for further service. From the second server either he/she may go outside the system or back to the first lower ordered server or may go to the third highest ordered server for further service depending upon the need of customer. From the third highest ordered server he/she may go outside the system or to the second server or to the first server.

Here, in this present paper, I have calculated waiting time W from mean queue length L, obtained by me i. e. Santosh Kumari (2021), by using the Littles Formula.

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Notations

λ_1: Mean Arrival rate at 1st server (S_1)
λ_2: Mean Arrival rate at 2nd server (S_2)
μ_1: Mean service rate of 1st server (S_1)
μ_2: Mean service rate of 2ndserver (S_2)
μ_3: Mean service rate of 3rd server. (S_3)
μ_4: Mean service rate of 4th server. (S_4)
μ_5: Mean service rate of 5th server. (S_5)
μ_6: Mean service rate of 6th server. (S_6)
P_{ij}: the probability of customer going from ith to jth server
P_i: the probability of customer going from ith to outside the whole queueing system

$P_{n1,n2,n3,n4,n5,n6}$: Probability of having $n1,n2,n3,n4,n5$ and $n6$ customers at S_1, S_2, S_3, S_4, S_5 and S_6 respectively.

If we take L as the mean queue length then the waiting time of the customer in the system is given by W:

$$L=\frac{-\lambda_1}{\lambda_1-\mu_1}-\frac{\lambda_2}{\lambda_2-\mu_2}+\frac{\lambda_1+\lambda_2-\mu_1-\mu_2}{\mu_1}+\frac{(\lambda_1+\lambda_2)p_{34}}{(\mu_4-\mu_3 p_{34})}-\frac{(\lambda_1+\lambda_2)p_{35}}{p_{35}\mu_3-\mu_5}-\frac{(\lambda_1+\lambda_2)p_{36}}{(\mu_3 p_{36}-\mu_6)}$$

$$W=\frac{L}{\lambda_1+\lambda_2}$$

Numerical Results and Discussion

- Behaviour of waiting time (W) of customer in the system with respect to arrival rate of first server (λ_1) for different values of arrival rate of second server (λ_2) is depicted in Table 1 and in Fig. 1 keeping the values of other parameters as fixed.

Table 1

$\mu_1 = 15$, $\mu_2 = 16$, $\mu_3 = 17$, $\mu_4 = 18$, $\mu_5 = 19$, $\mu_6 = 20$, $p_{34}=0.6$, $p_{35}=0.3$, $p_{36}=0.1$			
	$\lambda_2=12$	$\lambda_2=13$	$\lambda_2=14$
λ_1	W	W	W
1	0.247926336	0.337643761	0.504287752
2	0.248292636	0.33200448	0.488585676
3	0.249525847	0.327928625	0.475538919
4	0.251697564	0.325360666	0.464913053
5	0.254950684	0.324340662	0.456601871
6	0.259525847	0.325022923	0.450636958
7	0.265812397	0.327720292	0.44722426
8	0.274446482	0.332995235	0.446827434
9	0.286509974	0.341849079	0.450347103
10	0.303970292	0.356144205	0.459525847

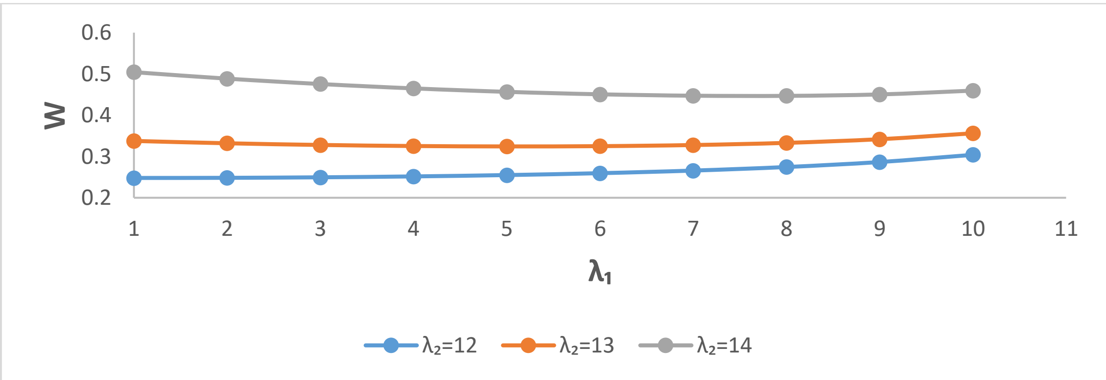

Fig. 1

Following can be interpreted from Table 1 and Fig. 1:

- Waiting time get increased with the increase in λ_1 for λ_2=12 but decreases with increase in in λ_1 for λ_2=13 and λ_2=14.
- Waiting time increases with respect to increase in λ_1 .

- Behaviour of the waiting time (W) of customer in the system w.r.t. μ_1 for different values of μ_2 is depicted in Table 2 and Fig. 2 keeping the values of other parameters fixed shown therein,

Table 2

λ_1=, 10, λ_2=, 12,μ_3= 17,μ_4 = 18, μ_5 = 19, μ_6= 20, p_{34}=0.6, p_{35}=0.3, p_{36}=0.1			
	μ_2=16	μ_2=17	μ_2=18
μ_1	W	W	W
15	0.303970292	0.273667261	0.25245514
16	0.287682413	0.257568776	0.236546049
17	0.275857228	0.245910704	0.225055089
18	0.266849079	0.2370511	0.216344029
19	0.2597385	0.230073428	0.209499266
20	0.253970292	0.224424837	0.203970292
21	0.24918871	0.21975148	0.19940516
22	0.245154865	0.215816022	0.195568088
23	0.241702125	0.212453113	0.19229501
24	0.238710551	0.209543885	0.189468127

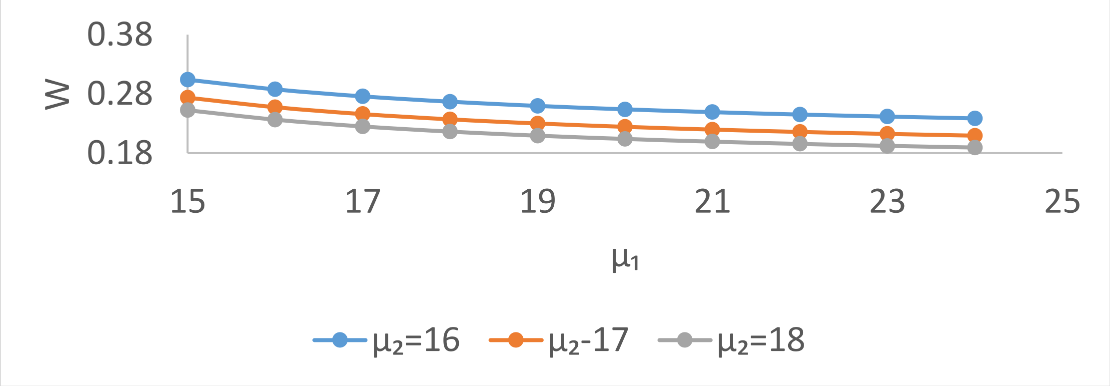

Fig. 2

From the above Table 2 and Fig. 2, it may be observed that the waiting time W decreases with the increase in service rate of first server (μ_1) as well as increase in the value of service tate of second server μ_2.

- Behaviour of the waiting time (W) of customer in the system w.r.t. μ_3 for different values of μ_4 is depicted in Table 3 and Fig. 3 keeping the values of other parameters fixed shown therein.

Table 3

λ_1=, 10, λ_2=, 12,μ_1 = 15, μ_2 = 16, μ_5 = 19, μ_6= 20, p_{34}=0.6, p_{35}=0.3, p_{36}=0.1			
	μ_4=16	μ_4-17	μ_4=18
μ_3	W	W	W
17	0.330495491	0.315282509	0.303970292
18	0.342937944	0.324327523	0.310886662
19	0.358516035	0.33522411	0.318990344
20	0.378632479	0.348632479	0.328632479
21	0.405679228	0.365572276	0.340319751
22	0.44409724	0.387706263	0.354811526
23	0.503170379	0.417943106	0.373300249
24	0.606105547	0.461874778	0.397772214
25	0.831801242	0.531801242	0.431801242
26	1.732532841	0.661104269	0.482532841

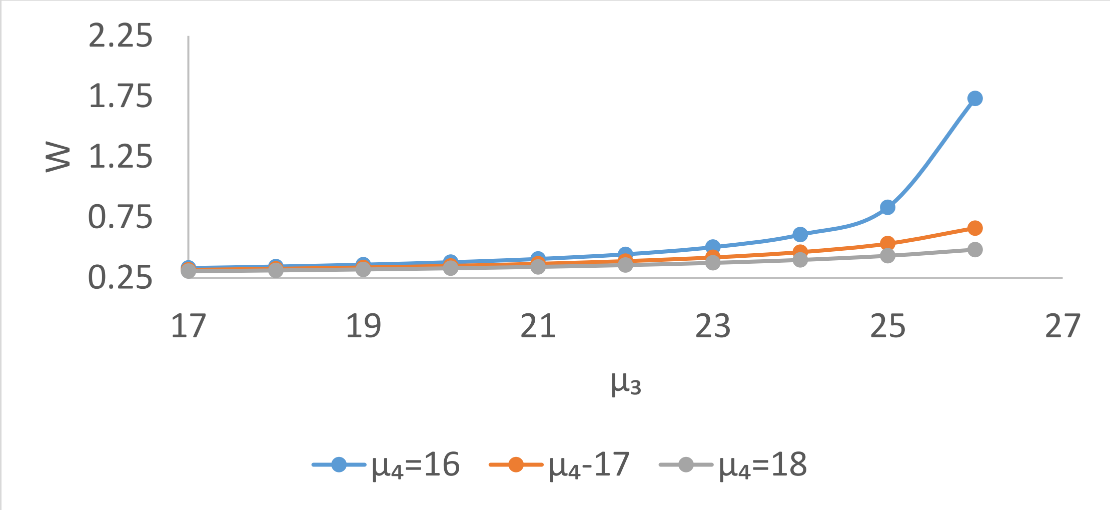

Fig. 3

Following can be interpreted from Table 3 and Fig. 3.

- Waiting time get increased with the increase in μ_3.
- Waiting time decreases with increase in μ_4.

- Behaviour of the waiting time (W) of customer in the system w.r.t. μ_5 for different values of μ_6 is depicted in Table 4 and Fig. 4 keeping the values of other parameters fixed shown therein.

Table 4

λ_1=, 10, λ_2=, 12, μ_1 = 15, μ_2 = 16, μ_3= 17, μ_4= 14, p_{34}=0.6, p_{35}=0.3, p_{36}=0.1			
μ_5	μ_6=20	μ_6=25	μ_6=30
	W	W	W
17	0.388569302	0.387396666	0.38663839
18	0.386615032	0.385442396	0.38468412
19	0.384941952	0.383769316	0.38301104
20	0.383493446	0.382320811	0.381562534
21	0.382227142	0.381054507	0.38029623
22	0.381110697	0.379938062	0.379179785
23	0.380118994	0.378946359	0.378188082
24	0.379232234	0.378059598	0.377301322
25	0.378434595	0.377261959	0.376503683
26	0.377713285	0.376540649	0.375782373

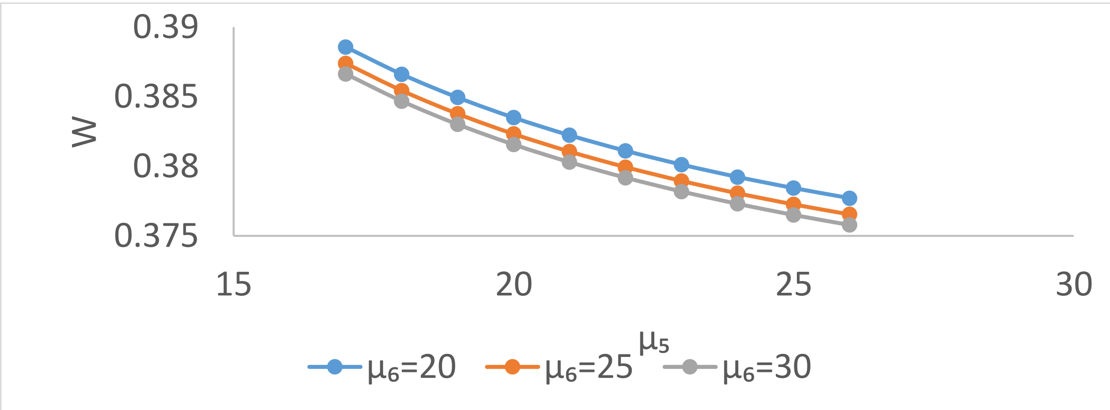

Fig. 5

Following can be interpreted from Table 4 and Fig. 4.

- Waiting time get decreased with the increase in μ_5 .
- Waiting time decreases with increase in $\mu_{6.}$

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