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# ALTERNATIVE STOCK MARKET MODELS FOR BSE AND NSE INDICES

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**ABSTRACT**

*The sudden and large deviations from normal pricing trend in the financial markets, often called jumps, have been the subject of much research. Attempts have been made to model these volatilities using Jump diffusion processes. However, the extent of fit of these models leaves much scope for modifications to arrive at a better mechanism that can be used to model multiple asset classes or multiple indices within the same asset class. In this project, using Hurst coefficient on BSE data, we showed that there is long range dependence in the stock market, which essentially means that the current stock price follows a trend as done in the past. This was also validated using the NIFTY index, which showed similar persistence in returns as the BSE SENSEX and indicated the presence of long-range dependence in returns. We also studied the application of fractional Brownian motion as a possible to alternative to model stock market returns.*

***KEYWORDS:*** *NSE, BSE, NIFTY, Stock Market, Black-Scholes model, Jump Diffusion, Hurst coefficient.*

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## Introduction

The sudden and large deviations from normal pricing trend in the financial markets, often called jumps, have been the subject of much research. Attempts have been made to model these volatilities using Jump diffusion processes [see Piazzesi (2005), Lee and Mykland (2008), and Tauchen and Zhou (2005)]. However, the extent of fit of these models leaves much scope for modifications to arrive at a better mechanism that can be used to model multiple asset classes or multiple indices within the same asset class. In this context, attempts made at modeling the stock market indices often classify the movement as a continuous time stochastic process with a drift, superimposed with random fluctuations. However, any large abrupt deviations from the trend warrant the inclusion of additional parameters in the model to closely approximate jumps. The arrival of jumps is often regarded to follow a Poisson distribution. However, jump diffusion models suffer from a major shortcoming in that the volatility clustering effects are not captured. In studies done so far, attempt had been made to model the Bombay Stock Exchange's Sensitivity Index using the Black-Scholes model. The model methodologically proved that inclusion of jumps would mitigate some of the non-idealities observed due to deviation from ideal fit using Black-Scholes model. Two jump diffusion based models viz. model incorporating normally distributed jump amplitudes and another one incorporates double exponential distributed jump amplitudes. Multinomial maximum likelihood estimation technique given by Hanson and Synowiec were used for parameter estimation.

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In this project, we furthered the study and generated some insights into the changes occurring in the stock indices. Using Hurst coefficient on BSE data, we showed that there is long range dependence in the stock market, which essentially means that the current stock price follows a trend as done in the past. This was also validated using the NIFTY index, which showed similar persistence in returns as the BSE SENSEX and indicated the presence of long-range dependence in returns.

**Mathematical Formulations**

The Black-Sholes model assumes that the stock prices/market indices are governed by Geometric Brownian Motion (GBM) diffusion process, governed by following equation:

$$dY = Y(t)(\mu dt + \sigma dw)$$

Where,

Y(t) is the stock price/index at time t

μ represents drift

σ represents volatility

W(t) is a Weiner Process ~ N(0,1)

- To take jumps into account, an addition term is added to Black-Scholes model.

$$dY\ t\ = Y\ t\ \ \mu dt + \sigma dW\ \ + Y\ t\ J\ q\ dN_t(t)$$

Where,

J(Q) represents the random jump amplitude

N(t) is discontinuous and one dimensional Poisson process

- Jump diffusion model with normally distributed jump amplitudes

This model was originally introduced by Merton in 1976. Here Q is assumed to be normally distributed with following normal density function:

$$f_Q\ x\ = \varphi(x,\mu,\sigma_j{}^2) = \frac{1}{2\pi\sigma_j{}^2}\exp\left(-\frac{(x-\mu)^2}{2\sigma_j{}^2}\right)$$

Where,

μ is mean of normal distribution and

$\sigma^2$ is variance of normal distribution

- Jump diffusion model with double exponential distribution jump amplitude

Double exponential model was presented by Kou in 2002. Q has following distribution:

$$f_Q\ x\ = p\eta_1 \exp\ -\eta_1 x\ \ 1_{\ 0,\infty}\ \ x\ + q\eta_2 \exp\ (\eta_2 x) 1_{\ -\infty,0}$$

Where $\eta_1 > 1, \eta_2 > 0, p \geq 0, q \geq 0, and p + q = 1$

**Jump Based BS Model**

Black-Scholes model does not adequately account for the large jumps (also called the tail events) that are observed more often than not in the financial markets. Due to the presence of these jumps, the plot of log returns show thicker edges as compared to a normal distribution. Movement in any market's stock index (movement in level of BSE for instance) is defined as weighted average price movement in the representative set of stocks listed in that stock exchange. A jump in a stock market index can be triggered by a significant price movement in one of the heavy-weight stock. Furthermore, there are many underlying reasons, systemic and un-systemic, which can cause significant move in stock price. To rectify the log-normal distribution assumption considered in the BS model, jumps are introduced in the equation which can account for the deviation between a log-return and normal distribution. Next section discuses technique to identify presence of jumps in a distribution.

**Testing for Jumps - Bipolar Variation Test**

Bipolar variation test proposed by Barndorff-Nielsen and Shephard (BNS) is used to detect the presence of financial time series data. The test at fundamental level makes use of the fact that a quadratic variation process of a semi-martingale can be distributed into two parts – a continuous part and a discontinuous part. This gives us the following formulations:

Here Y, the log return of stock market index, is assumed to be a martingale and $Y^c$ and $Y^d$ represent the continuous and discontinuous part of the local martingale component of Y, respectively. The quadratic variation process of Y can be defined by:

$$[Y]_t = p - \lim_{n\to\infty} \sum_{j=0}^{n-1} (Y_{t_{j+1}} - Y_{t_j})^2$$

For a sequence of partitions $t_0 = 0 < t_1 < t_2 < \ldots < t_n = t$ such that $\sup_j \{t_{j+1} - t_j\} \to 0$ and $n \to \infty$. Then following identity holds:

$$[Y]_t = [Y^c]_t + [Y^d]_t$$

$$\text{Where } [Y^d]_t = \sum_{0<u<t} \Delta Y_u^2 \text{ where } \Delta Y_t = Y_t - Y_{t-} \text{ are jumps in } Y.$$

Now, to check for jumps, following hypothesis testing is done:

**$H_0$** : *Y belongs to Brownian Semi-martingale class*

$$Y_t = \int_0^t a_s ds + \int_0^t \sigma_s dW_s$$

**$H_1$** : *Y belongs to Brownian Semi-martingale plus jumps class*

$$Y_t = \int_0^t a_s ds + \int_0^t \sigma_s dW_s + \sum_{j=1}^{N_t} c_j$$

Here, $a_s$ and $c_j$ are càdlàg.

Null hypothesis in the above BV test states that the process does not contain jumps. Therefore, rejection of null hypothesis will indicate presence of jumps in the process. BNS proposed following test statistics to perform hypothesis testing:

$$\hat{G} = \frac{\delta^{-1/2}(\mu_1^{-2}\{Y_\delta\}_t^{[1,1]} - [Y_\delta]_t}{\sqrt{\mu_1^{\,4}\{Y_\delta\}_t^{[1,1,1,1]}}}$$

$$\hat{H} = \frac{\delta^{\,1/2}\left(\frac{\mu_1^{-2}\{Y_\delta\}_t^{[1,1]}}{\{Y_\delta\}_t} - 1\right)}{\sqrt{\{Y_\delta\}_t^{[1,1,1,1]} - 1/\{\{Y_\delta\}_t^{[1,1]}\}^2}}$$

$$\hat{J} = \frac{\delta^{-1/2}\left(\frac{\mu_1^{-2}\{Y_\delta\}_t^{[1,1]}}{\{Y_\delta\}_t} - 1\right)}{\sqrt{\max(t^{-1}, \{Y_\delta\}_t^{[1,1,1,1]} \quad 1/\{\{Y_\delta\}_t^{[1,1]}\}^2)}}$$

Here,

$$[Y_\delta]_t = \sum_{j=1}^{\lfloor t/\delta \rfloor} y_j^2 \text{ , } \{Y_\delta\}_t^{[1,1]} = \sum_{j-2}^{\lfloor t/\delta \rfloor} |y_{j-1}||y_j| \text{ , } \{Y_\delta\}_t^{[1,1,1,1]} = \frac{1}{\delta} \sum_{j-4}^{\lfloor t/\delta \rfloor} y_{j-4}||y_{j-3}||y_{j-2}|y_{j-1}|$$

And $\mu_1 = \sqrt{\frac{2}{\pi}}$

$Y_i$ represent log-returns. $\delta$ is taken to be 1 as daily index level data is used.

**Jump Diffusion Processes**

To model jump diffusion processes, there are two aspects of the distribution which are critical. First in the discrete occurrence of jumps and second is amplitude of jumps. Many studies have consistently used Poisson processes to model jump arrival times. However, several models exist to incorporate variation in jump amplitudes. Most popular of these methods assume – Normal and Double exponential distribution of jump amplitudes. Mathematical formulation of jump diffusion model appends an additional term to the Black-Sholes model. Added term accounts for jumps in the stock price/index.

$$dY(t) = Y(t)(\mu dt + \sigma dW\ ) + Y(t)J(Q)dN_t(t)$$

Here,

J(Q) = Random jump amplitude

$N_t(t)$ = Discontinuous and one dimensional Poisson process with constant jump rate

$$P(N_t = k) = \frac{(\lambda t)^k}{k!} e^{\ \lambda t}$$

Above equation shows a Poisson process to model jumps with constant jump rate, λ.

$$J(Q)dN(t) = \sum_{i=1}^{dN(t)} J(Q_i)$$

*J(Q)dNt(t)* is a symbol used to define sum of *$dN_t(t)$* jumps of a compound Poisson process.

Lastly, on transforming the stock price/index stochastic differential euqation (SDE) to the SDE of instantaneous log returns using stochastic chain rule for Markov Processes in continuous time we obtain:

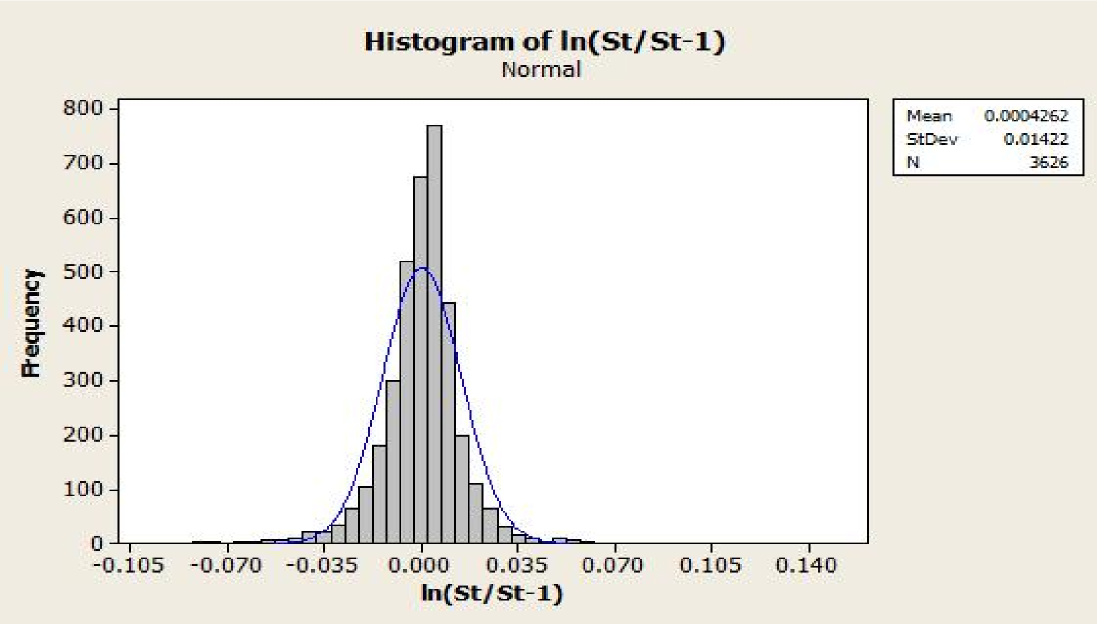
$$d(\ln(Y_t)) = (\mu - \frac{\sigma^2}{2})t + \sigma dW_t + QdN_t$$

**Long Range Dependence in Indian Stock Market**

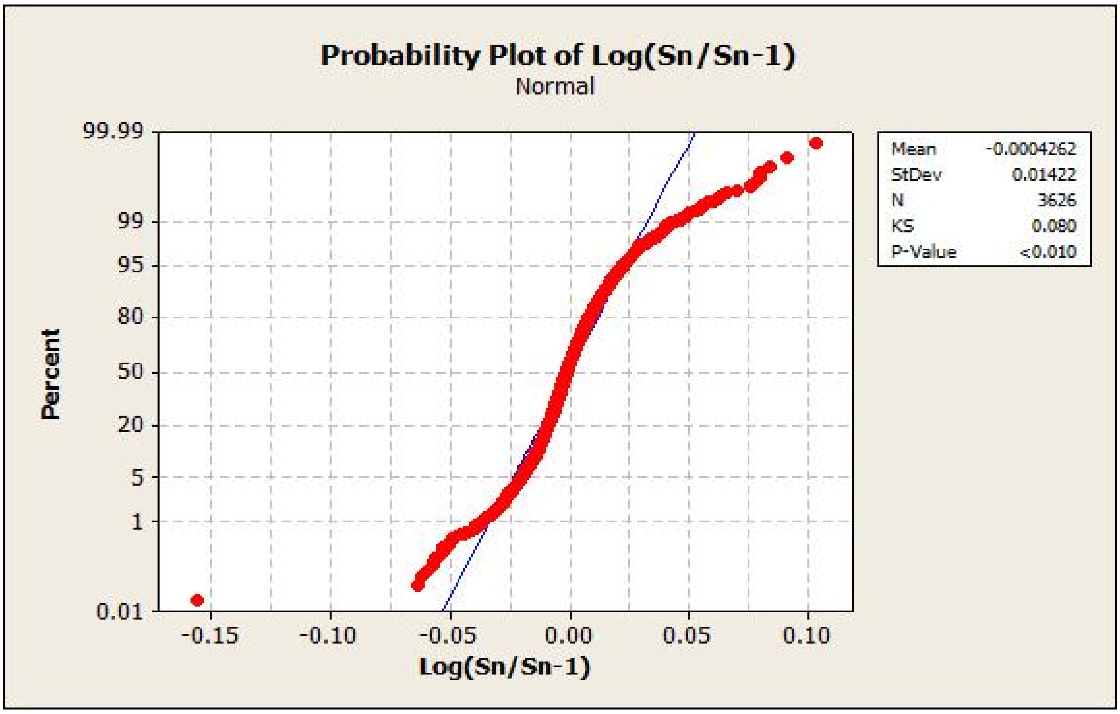
The concept of Efficient Markets entails that any new piece of information would rapidly adjust security prices in an efficient capital market. This essentially means that the current prices of assets reflect all information about the asset. The price adjustments may, however, be imperfect, in the sense that the prices may sometimes over-adjust or under-adjust. The adjustment however cannot be predicted by an investor and therefore, the only difference between prices $P_t$ and $P_{t-1}$ are events that we cannot possibly predict, or a random event. As a result, the stock prices should follow a random walk hypothesis in an efficient market. This leads to the argument that the data of stock prices and returns should follow normal distribution with stable mean and finite variance. This approach means that information arrives to an investor linearly and reaction of an investor to 'linear' information is instantaneous. This is based on the assumption that yesterday's information has been already folded into yesterday's price.

To test the random walk hypothesis, we collected daily data for BSE SENSEX and NSE NIFTY indices between 3rd January, 2000 and 15th July 2014. The collected data for daily close of BSE and NSE indices were checked for normality assumption. The results showed that data from both indices did not fit well with normal distribution as it had higher peak than theoretically predicted around the mean and fatter tails. The presence of fatter tails indicates 'memory' effects which arise due to nonlinear stochastic processes. Actually the information flow to an investor is clustered and its arrival is irregular rather than continuous and smooth in nature. This clustered and/or irregular arrival of 'new' information results in periods of low and high volatility which results in 'leptokurtic' distribution instead of normal.

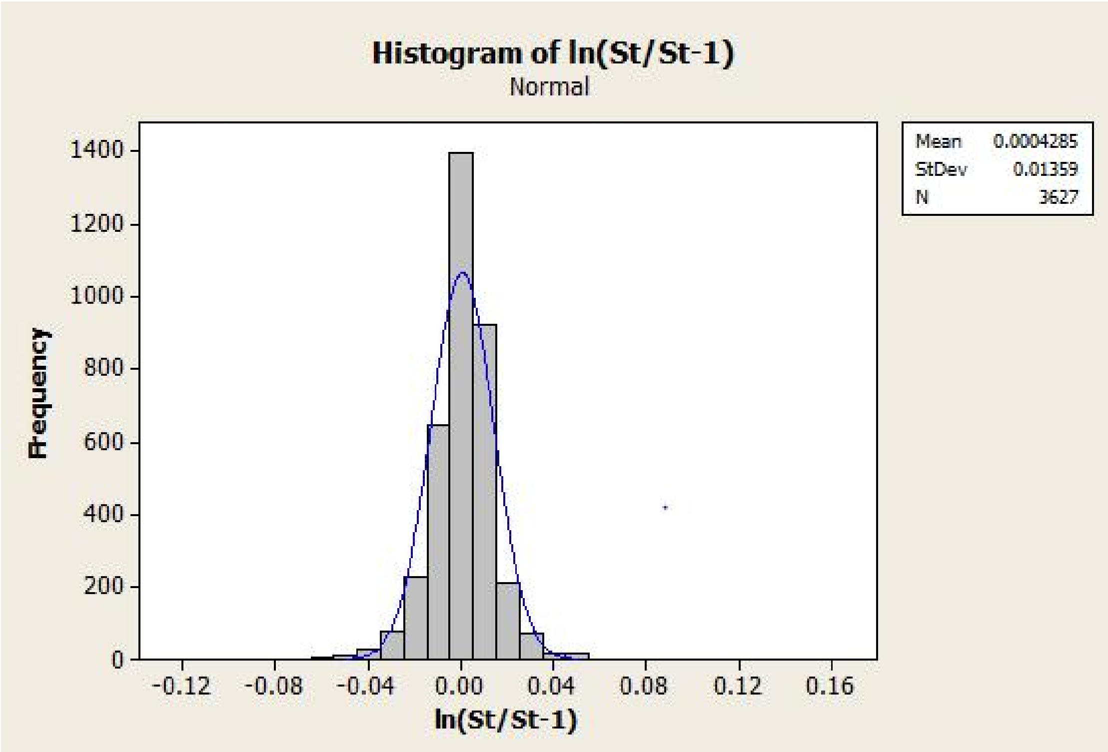
**Log Returns from BSE SENSEX between 03/01/2000 and 15/05/2014 is shown below:**




We apply the **Kolmogorov-Smirnov test** in Minitab to test normality in the log-return data. At the 5% significance level, the test rejectsthe Null Hypothesis of the data coming from a normal distribution.



**Log Returns from NSE NIFTY between 03/01/2000 and 15/05/2014 is shown in the figure below:**

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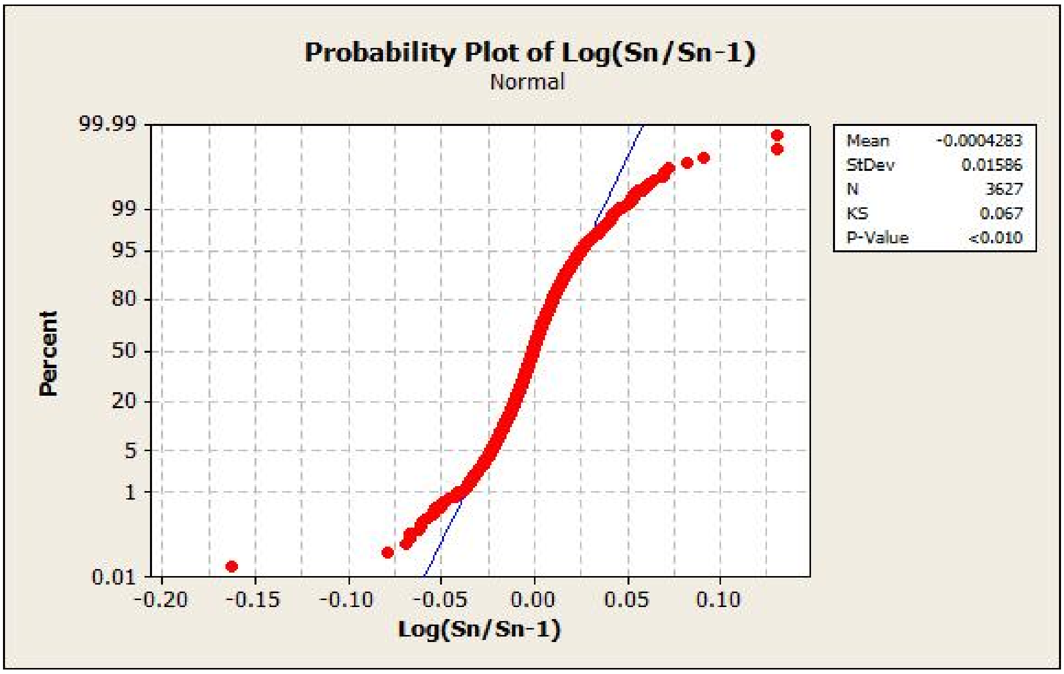

**Hurst Exponent–Persistence of Returns**

Hurst (1951) introduced a measure of persistence in data set through calculation of Hurst exponent

$$H = \log (R/S) / \log(T/2)$$

Let z(T) be the mean of a time series over a chosen period of time T.

$$z(T) = (\frac{1}{T}) \sum_{t=1}^{T} Z(t)$$

Let $X(t)$ be the accumulated departure of the influx $Z(t)$ from the mean z(T)

$$X(t,T) = \sum_{u=1}^{t} (z(u) - z(T))$$

The range R is then defined as the difference between the maximum and the minimum accumulated influx X:

$$R(T) = \max\big(X(t,T)\big) - \min\,(X(t,T))$$

The standard deviation S is given by

$$S = [\frac{1}{T} \sum_{n=1}^{\infty} (Z(t) - z(T))\hat{}2\,]\hat{}(\frac{1}{2})$$

The ratio R/S is given by

$$R/S = (T/2)^H$$

Where H is the Hurst exponent. There are three scenarios that describe the behavior of data series based on the value of H

1. H = 0.5

H equals 0.5 denotes a random series or white noise.

2. 0 ≤ H < 0.5

This denotes an anti-persistent or mean-reverting system. This essentially means that an upward movement in one period is likely to be followed by a downward movement in the next period and vice versa.

3. 0.5 < H ≤ 1.0

If the value of H lies between 0.5 and 1.0, the series is essentially trend reinforcing or persistent. This essentially means that an upward (downward) movement in one period is likely to be followed by an upward (downward) movement in the next period.

For the BSE SENSEX daily data between 3rd January 2000 and 15th July, 2014, we used MATLAB simulations to find out the Hurst coefficient for our time series data. The value of parameter H came out to be 0.635 which shows significant persistent behavior in returns from BSE SENSEX within the given period. As this value lies between 0.5 and 1.0, we concluded the presence of long range memory behavior in returns from the stock index.

For NIFTY, the value of Hurst coefficient came out to be 0.648, showing a similar persistence in returns as the BSE SENSEX and presence of long-range dependence in returns from the NIFTY index.

The results provided conclusive evidence against the Efficient Market hypothesis and suggest Fractional Brownian Processes to provide a good fit for the returns from BSE and NSE indices as fractional Brownian motion captures this long range behavior.

**Fractional Brownian Motion and ARFIMA Processes**

One of the first long-term memory process, called the Fractional Brownian Motion, was defined by Mandelbrot and Van Ness (1968). The Hurst exponent, H, forms the key parameter of such processes, which characterizes the self-affinity of FBM.

Fractional Brownian motion, denoted by $B_H(t,\omega)$, is a zero-mean Gaussian stationary process, defined as:

$$B_H(t,\omega)\text{-}B_H(0,\omega)=\frac{1}{\Gamma(H+\frac{1}{2})}\left\{ \begin{array}{c} \int_{-\infty}^{0}\left[(t-s)^{H-\frac{1}{2}}-(-s)^{H-\frac{1}{2}}\right]dB(s,\omega) \\ +\int_{0}^{t}(t-s)^{H-\frac{1}{2}}dB(s,\omega) \end{array} \right\}$$

Where $b_0 = B_H(0,\omega)$ . If $b_0 = 0$, the equation reduces to:

$$B_H(t,\omega) = \frac{1}{\Gamma(H+\frac{1}{2})}\left\{\int_0^t (t-s)^{H-\frac{1}{2}} dB(s,\omega)\right\}$$

Hence, if H = ½, fractional Brownian motion reduces to ordinary Brownian motion and becomes a short-term memory process.

ARFIMA processes form another class of discrete time models linked to FBM. These models are a generic form of the standard ARIMA(p,d,q) processes for which the differencing parameter d was an integer. Strictly speaking, a non-stationary time series is modeled by ARIMA(p,d,q) process, where d = 1. On the other hand, a stationary series is modeled by ARMA(p,q) process, where d = 0. ARIMA processes refer to infinite memory and ARMA processes represent short-term memory. Therefore, ARFIMA (p,d,q) forms a mid-way between the two where d is a fraction. The relationship between H and d is given by:

$$d = H - 1/2$$

Using the values of H we obtained for SENSEX and NIFTY, we calculated the value of d as

$$d_{SENSEX} = 0.635 – 0.5 = 0.135$$

$$d_{NIFTY} = 0.648 – 0.5 = 0.148$$

**Conclusions**

- **Long Range Dependence in Indian Markets :** The results showed that data from both indices did not fit well with normal distribution as it had higher peak than theoretically predicted around the mean and fatter tails. The presence of fatter tails indicates 'memory' effects which arise due to nonlinear stochastic processes. Actually the information flow to an investor is clustered and its arrival is irregular rather than continuous and smooth in nature. This clustered and/or irregular arrival of 'new' information results in periods of low and high volatility which results in 'leptokurtic' distribution instead of normal.
- **Hurst Exponent:** The value of parameter H came out to be 0.635 which shows significant persistent behavior in returns from BSE SENSEX within the given period. As this value lies between 0.5 and 1.0, we concluded the presence of long range memory behavior in returns from the stock index. For NIFTY, the value of Hurst coefficient came out to be 0.648, showing a similar persistence in returns as the BSE SENSEX and presence of long-range dependence in returns from the NIFTY index.

The results provided conclusive evidence against the Efficient Market hypothesis and suggest Fractional Brownian Processes to provide a good fit for the returns from BSE and NSE indices as Fractional Brownian motion captures this long range behavior.

**References**


~ Suzanne S. Lee, Per A. Mykland: Jumps in Financial Markets: A New Nonparametric Test and Jump Dynamics 2008, The Review of Financial Studies v 21 n 6 2008

~ Jan Hanousek, EvzenKocenda, Jan Novotny: Price jumps on European stock markets, 2013, J. Hanousek et al. / Borsal_stanbul Review xx (2013) 1e13

~ Black, Fischer; Myron Scholes (1973). "The Pricing of Options and Corporate Liabilities". Journal of Political Economy 81(3): 637–654

~ Tauchen, G., and H. Zhou. 2005. Identifying Realized Jumps on Financial Markets. Working Paper, Department of Economics, Duke University.

~ Gilbert Abraham-Frois, 1998. Non-Linear Dynamics and Endogenous Cycles.

~ Alvaro Cartea, Diego del-Castillo-Negrete,2006. Fractional diffusion models of option prices inmarkets with jumps

~ Yasuyoshi Tokutsu, Shuichi Nagata and Koichi Maekawa, 2008. A Comparison of Estimators for Long-Memory Process-Simulation and Empirical Study.

~ Javier E. Contreras-Reyes, 2012. Statistical Analysis of Autoregressive FractionallyIntegrated Moving Average Models.


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